Intertemporal Distortions in the Second Best

Appendix

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August 30, 2011

We show how several benchmark examples fit into the general framework for second best problem presented in the main text. We cover several versions of the Ramsey model, self-enforcement constraints, private information, limited commitment, and principal-agent settings. In each case we explain the mapping between the particular admissibility constraints and our general formulation. We also discuss standard assumptions for each model class that map into our Assumptions 6 to 8. To keep the text concise we refer the reader to the literature for a derivation and discussion of the admissibility constraints in each environment.

1 Ramsey Models

We start with the Ramsey model of optimal taxation. Chari and Kehoe (1999) review the macroeconomic applications of this paradigm.

We begin with the simplest version with a representative agent and no uncertainty. We then consider aggregate shocks, agent heterogeneity, and incomplete markets.

1.1 Representative agent and no uncertainty

The main assumption in Ramsey models is that lump sum taxes are not available to the government in order to finance an exogenously given sequence of spending. The underlying economy is the standard neoclassical model. For now we abstract from aggregate or idiosyncratic shocks. Thus time summarizes the exogenous state $s^t = t$.

The restrictions on the tax instruments are captured by an implementability constraint upon allocations at date $t = 0$:

$$\sum_{t=0}^{\infty} \beta^t \left(u_t^c c_t + u_t^l l_t \right) \geq u_0^c \left\{ \left[\left(1 - \tau_0^k \right) r_0 + 1 - \delta \right] k_0 + b_0 \right\}.$$

The implementability constraint can be derived from the equilibrium optimality conditions: See Chari and Kehoe (1999) for a complete discussion. Since the implementability constraint summarizes *all* the restrictions on the government's choice of allocations, it defines the set of admissible allocations.

Now we detail how to map the implementability constraint into our general framework. Since there are no admissibility constraints at later dates, we can drop the dependence on the state s^t. Let

$$h^0(x_t) = -\left(u_t^c c_t + u_t^l l_t \right)$$

and

$$\mathbf{h}^0(\psi) = \sum_{t=0}^{\infty} \beta^t h^0(x_t).$$

Thus the term d^0 is trivially set to 1. Similarly, we set $\mathbf{b}_{m,i}\left(\psi, s^t\right) = 0$ and $\mathbf{h}^1_{m,i}\left(\psi, \tilde{s}^t\right) = 0$. The initial value for the auxiliary variable is set at

$$a_0 = u_0^c \left\{ \left[\left(1 - \tau_0^k\right) F_0^k \right] k_0 + b_0 \right\}.$$

Typically, the value of assets a_0 is exogenously restricted to exclude a solution in which the government sets τ_0^k high enough to pay all outstanding debt and raise enough assets such that no distortionary taxes are ever needed. Thus the implementability constraint is captured as

$$H(\psi) = \mathbf{h}^0(\psi) \leq a_0.$$

There is no need to specify the law of motion for the auxiliary variable.

1.2 Aggregate shocks

Consider a stochastic economy where aggregate state z_t determines productivity or government spending as in Zhu (1992). The exogenous state of the economy now is $s^t = z^t$. As long as the government has access to the full array of state-contingent claims, a single implementability constraint at date $t = 0$ remains sufficient to summarize all the constraints on allocations in a Ramsey equilibrium:

$$\sum_{t=0}^{\infty} \sum_{z^t} \pi(z^t)\beta^t \left(u_t^c(z^t)c(z^t) + u_t^l(z^t)l(z^t) \right) \geq u_0^c \left\{ \left[\left(1 - \tau_0^k\right) r_0 + 1 - \delta \right] k_0 + b_0 \right\}.$$

The mapping to our framework is essentially unchanged from the previous case. Let

$$h^0(x(z^t)) = u_t^c(z^t)c(z^t) + u_t^l(z^t)l(z^t)$$

as before. Now the term $\mathbf{h}^0$ takes the form

$$\mathbf{h}^0(\psi) = \sum_{t=0}^{\infty} \sum_{z^t} \pi(z^t)\beta^t h^0(x(z^t)).$$

The rest of steps are identical from the case without aggregate shocks.

1.3 Agent heterogeneity

Next we briefly consider a Ramsey model with heterogeneous agents. For simplicity, consider two ex-ante types, $i = 1, 2$, that differ in their preferences over leisure, and abstract from aggregate or idiosyncratic shocks. In addition to the implementability constraints, a competitive equilibrium also requires that agents equate their marginal rate of substitution between consumption and leisure,

$$\frac{u_{1t}^l}{u_{1t}^c} = \frac{u_{2t}^l}{u_{2t}^c},$$

at all dates $t \geq 0$. We thus now need to map a sequence of constraints. Let

$$b(x_t) = \frac{u_{1t}^l}{u_{1t}^c} - \frac{u_{2t}^l}{u_{2t}^c}.$$

(Recall x_t is the distribution of consumption and leisure allocations across agents). For this admissibility constraint, there is no need for any additional term, and the auxiliary variable is trivially set to 0 at all dates. Thus

$$H_1(\psi, t) = b(x_t) \leq 0$$

for all $t \geq 0$. If the admissibility constraint can be binding in either direction, then it is necessary to include $H_2(\psi, t) = -b(x_t) \leq 0$ as well.

As long as we retain the assumption of complete markets, it is also possible to encompass economies with idiosyncratic shocks, or to combine the heterogeneous agents model with aggregate shocks.

1.4 Incomplete markets

We have maintained the assumption of complete markets in our previous examples. We now consider the case where only a risk-free bond is traded. The incomplete markets Ramsey model has been analyzed by Aiyagari, Marcet, Sargent and Seppala (2002) and Farhi (2010). Here the role of the auxiliary variable and its law of motion become clear. We abstract from agent heterogeneity, and focus on aggregate shocks. Thus $s^t = z^t$.

The economy is similar to the simple Ramsey model. There are aggregate shocks but bond returns cannot be contingent on the state of the economy, so that markets are incomplete. Formally, bond repayments at time t are not measurable with respect to s^t and only depend on s^{t-1}, and will be denoted with $B_t\left(s^{t-1}\right)$. In addition, the after-tax return to capital is not allowed to depend on the state s_t. This constraint is imposed in order to prevent the government from effectively completing markets.[1]

Manipulating the competitive equilibrium conditions and the government's present value budget constraint we obtain the implementability constraint at node s^t:

$$\frac{1}{u^c(s^t)} \sum_{j=t}^{\infty} \sum_{s^j \in S^j | s^t} \beta^{j-t} \pi\left(s^j | s_t\right) \left(h\left(x\left(s^j\right), s_j\right) - \tilde{T}\left(s^j\right)\right) = B\left(s^{t-1}\right) + k\left(s^{t-1}\right), \tag{1}$$

where

$$h\left(x\left(s^j\right), s_j\right) = u^c(s^j)c(s^j) + u^l(s^j)l(s^j),$$

and

$$\tilde{T}\left(s^j\right) = u^c(s^j)T\left(s^j\right).$$

The implementability constraint (1) must hold at all nodes. The government also faces the constraint that transfers must be non-negative:

$$\tilde{T}\left(s^t\right) \geq 0, \tag{2}$$

at all nodes.

To adapt the implementability constraints to our general formulation, first we define

$$b(k(s^{t-1})) = -k(s^{t-1})$$

and

$$d(x(s^t)) = \frac{1}{u^c(s^t)}.$$

[1] Farhi (2010) restricts the capital tax instead of the return as he approaches the Ramsey equilibrium specified in terms of taxes rather than allocations.

These terms do not depend directly on the exogenous state of the economy s_t. Then let

$$\mathbf{h}_1^0(\psi, s^t) = d(x(s^t)) \sum_{j=t}^{\infty} \sum_{s^j \in S^j | s^t} \beta^{j-t} \pi\left(s^j | s_t\right) h\left(x\left(s^j\right), s_j\right).$$

and then set

$$H_1(\psi, s^t) = b(k(s^{t-1})) + \mathbf{h}_1^0(\psi, s^t)$$

Since the implementability constraint must hold with equality, we also define

$$H_2(\psi, s^t) = -H_1(\psi, s^t).$$

In order to capture the possibility of transfers as well as the non-measurability constraints, we now need to specify a law of motion for the auxiliary variables. First we define

$$V_t\left(s^t\right) = \sum_{j=t}^{\infty} \sum_{s^j \in S^j | s^t} \beta^{j-t} \pi\left(s^j | s_t\right) \tilde{T}\left(s^j\right).$$

The variable $V\left(s^t\right)$ corresponds to the present discounted value of transfers at node s^t. Constraint (**??**) requires:

$$V_t\left(s^t\right) \geq \beta V_t\left(s^{t+1}\right) \geq 0, \tag{3}$$

for all $s^t \in S^t, t \geq 0$.

The relationship between auxiliary variables and equilibrium variables is

$$a_t\left(s^t\right) = \begin{bmatrix} V_t\left(s^t\right) + B_t\left(s^{t-1}\right) \\ -V_t\left(s^t\right) - B_t\left(s^{t-1}\right) \\ b_{t+1}\left(s^t\right) \\ V_t\left(s^t\right) \end{bmatrix}.$$

The implementability constraint at each node is then

$$\begin{aligned} H_1\left(\psi, s^t\right) &\leq a_1\left(s^t\right), \\ H_2\left(\psi, s^t\right) &\leq a_2\left(s^t\right). \end{aligned}$$

The third and fourth entries in the vector $a(s^t)$ are used to specify the law of motion. Market incompleteness and the non-negativity of transfers (**??**) translate into constraints on the correspondence $\Gamma\left(a\left(s^t\right), s_t\right)$. The fact that (1) must hold with equality gives rise to the constraint:

$$a_1\left(s^{t+1}\right) = -a_2\left(s^{t+1}\right).$$

The non-measurability of bond returns requires:

$$a_1\left(s^{t+1}\right) = a_3\left(s^t\right) + a_4\left(s^{t+1}\right).$$

Finally, the non-negativity of transfers (**??**) leads to the constraint:

$$a_4\left(s^{t-1}\right) \geq \beta a_4\left(s^t\right) \geq 0.$$

Both Aiyagari, Marcet, Sargent and Seppala (2002) and Farhi (2010) additionally consider the possibility of ad-hoc bounds on debt. These can be captured by constraining the law of motion $\Gamma\left(a\left(s^t\right), s_t\right)$ further. For example, a lower bound on debt $b(s^t) \geq B$ is simply a constraint on entry $a_3(s^t)$.

1.5 Regularity assumptions in Ramsey models

We conclude by briefly discussing how the literature on Ramsey models deals with the requirements of interiority, non-convexity, and convergence. These relate directly to Assumptions 6 to 8, respectively, in the main text.

Assumption 6 Most Ramsey models work with specifications that satisfy the usual Inada conditions, ruling out corner solutions. One exception is the use of quasi-linear preferences of the form $c-v(l)$ (see Farhi (2010), and Aiyagari et al. (2002), among others). These preferences are used only for models with a single representative household, and the non-negativity constraint on consumption is trivially not binding in this case.

Assumption 7 The implementability constraint usually renders the set of admissible allocations non-convex. The classic Chamley-Judd result does not require the first order conditions to be sufficient, but most of the applied work does. Under some preference specifications, there exist parameter restrictions such that the first order conditions can be shown to be sufficient as well.[2] However, these restrictions are too strong for practical purposes and most of the literature resorts to numerically checking that the solution is a global maximum.

Assumption 8 Most work in Ramsey models assume convergence, following Judd (1985) and Chamley (1986). Two exceptions are Aiyagari et al. (2002) and Farhi (2010) for Ramsey models with incomplete markets and aggregate shocks. They are able to prove convergence for the case of quasi-linear preferences, but have to resort to numerical methods for any other specification. Hagedorn (2010) shows that cycles are optimal for some Ramsey economies without capital. See also Hassler et al. (2008), who find cycles are optimal for the special case of full depreciation.

2 Risk sharing

Next we consider models where the provision of insurance is hindered by incentive-compatibility constraints. We describe how to map admissibility constraints arising from self-enforcement and private information into our framework.

2.1 Self-enforcement

We start with a simple example of risk sharing with self-enforcement constraints. Agents face idiosyncratic labor productivity shocks, θ_t, for $t = 0, 1, 2, 3...$ In each period t, they are characterized by their history of productivity shocks, denoted with $\theta^t = \{\theta_0, \theta_1, ..., \theta_t\}$. Thus we have $s^t = \theta^t$ as there are no aggregate shocks.

Absent an enforcement technology, an agent may have an incentive to deviate from the risk-sharing arrangement. If she deviates, she has access to an outside option, with value $V_{out}(k_t; \theta_t)$. There are many formulations for the outside option that fit in our framework. For example, Kehoe and Levine (1993) consider a partial insurance economy where agents can default on their commitments, in which case they are excluded from insurance markets from then onwards.[3] Since agents lose all claims to assets, the outside option depends only on their type, $V^{aut}(s^t)$. The self-enforcement constraint for type s^t can be then captured as follows:

$$H_1(\psi, s^t) = b_1(s_t) + \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t h_1^0(x(s^t))$$

where

$$h_1^0(x_t) = -u(c_t, l_t)$$

[2] See Lucas and Stokey (1983).

[3] See also Kocherlakota (1996).

and

$$b_1(s_t) = V^{aut}(s_t)$$

is the value of autarky.

Our formulation also allows to consider the possibility that agents hold onto their capital holdings if they default. The outside option would then be a function of the state of the economy *and* the individual holdings of capital. The latter are captured by $y(s^{t-1})$, so we can specify the admissibility constraint as

$$H_1(\psi, s^t) = b_1(y(s^t), s_t) + \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t)\beta^t h_1^0(x(s^t))$$

where

$$h_1^0(x_t) = -u(c_t, l_t)$$

and

$$b_1(y(s^t), s_t) = V^{aut}(y(s^t), s_t)$$

is now the value of autarky when an agent retains the claims to asset $y(s^t)$. In the same fashion we can clearly encompass many formulations of the outside option V.

Note it is easy to combine self-enforcement constraints with restrictions on tax instruments. For example, the government may not be able to use agent-specific lump-sum taxes. Thus we have to stack an implementability constraint to the set of admissibility constraints, following the steps in Section 1. While the resulting problem may be very difficult to solve, our results allow to easily determine whether there will be intertemporal distortions in the long run.

2.2 Private information

We now turn to private information economies. We concentrate on a canonical model in which each agent has an idiosyncratic preference θ over leisure and there are no aggregate shocks. Each agent's exogenous state is given by the history of idiosyncratic shocks $s^t = \theta^t$.

The current realization θ_t is only privately observable, which constraints the set of allocations that can be implemented. In particular, any allocation must induce truth-telling by the agents, giving rise to the incentive compatibility constraint corresponding to equation (5) in the main text. This constraint is intractable in most models. Thus, in applications, it is substituted by the following sequence of incentive compatibility constraints for all s^t:

$$\sum_{j=0}^{\infty} \sum_{s^{t+j} \in S^{t+j}|s^t} \beta^j \pi(s^{t+j}|s_t) u\left(x(s^{t+j}), \theta_{t+j}\right) \geq \sum_{j=0}^{\infty} \sum_{\tilde{s}^{t+j} \in S^{t+j}|s^t} \beta^j \pi(s^{t+j}|s_t) u\left(x(\tilde{s}^{t+j}), \theta_{t+j}\right) \tag{4}$$

for all possible deviations

$$\tilde{s}^{t+j} = \left\{s^{t-1}, \tilde{\theta}_t, \tilde{\theta}_{t+1}, \ldots, \tilde{\theta}_{t+j}\right\}$$

in $\tilde{\theta}_t \in \Theta$ and so on. Constraint (2) involves sequences of deviations from truth telling at each node, involving the entire continuation allocation. Under certain conditions (Spear and Srivastava, 1987), this constraint can be further simplified by considering only one period deviations at each node. Moreover, if the distribution of idiosyncratic shocks is i.i.d., imposing the single-crossing condition, allows to further simplify the incentive compatibility constraint, by considering only one type of deviations, namely of low disutility types reporting to be high disutility types. As a result, in most applications, there is only one incentive compatibility constraint per node s^t.

We now show how to express the incentive compatibility constraints (2) in our general framework. Let

$$h^0(x(s^{t+j});\theta_{t+j}) = -u\left(x(s^{t+j}),\theta_{t+j}\right)$$

and

$$\mathbf{h}^0(\psi,s^t) = \sum_{j=0}^{\infty} \sum_{s^{t+j}\in S^{t+j}|s^t} \beta^j \pi(s^{t+j}|s_t) h^0(x(s^{t+j});\theta_{t+j})$$

(thus $d^0(x(s^t);s_t) = 1$ trivially). To capture the terms associated with a deviation, we set

$$h^1(x(s^{t+j});\theta_{t+j}) = u\left(x(s^{t+j}),\theta_{t+j}\right)$$

and

$$\mathbf{h}^1(\psi,\tilde{s}^t) = \sum_{j=0}^{\infty} \sum_{\tilde{s}^{t+j}\in S^{t+j}|\tilde{s}^t} \beta^j \pi(s^{t+j}|s_t) h^1(x(\tilde{s}^{t+j});\theta_{t+j})$$

for the relevant deviation $\tilde{s}^t$. Finally, combining both terms

$$H(\psi,s^t) = \mathbf{h}^0(\psi,s^t) + \mathbf{h}^1(\psi,\tilde{s}^t) \le 0.$$

Term $\mathbf{b}$ is trivially set to zero, and the auxiliary variable is restricted to be equal to 0 at all states.

2.3 Regularity assumptions in risk sharing economies

Assumption 6 In self-enforcement economies, interiority is usually not an issue as the admissibility constraint itself prevents consumption from being too low, and most preference specifications rule out a zero labor choice. For private information economies, consumption and labor can be at corner solutions, unless Inada conditions hold. The possibility of zero consumption is related to the convergence properties of private information economies. It can be removed by imposing a lower bound on continuation utility, which also makes the economy stationary (Atkeson and Lucas, 1992). An additional constraint on continuation utilities can easily be incorporated in our framework as an admissibility constraint.

Assumption 7 Both self-enforcement and private information economies require additional conditions to ensure sufficiency. For economies with self-enforcement constraints, by imposing conditions on the outside option it is often possible to ensure that the set of admissible allocations is convex. For example, if the outside option is a function of the exogenous state of the economy (as in the case of autarky), the set of admissible allocations is convex.[4] For outside options that do depend on allocations, it is necessary to provide some additional restrictions.

For private information economies it is difficult to show that the set of admissible allocations is convex. A standard procedure is to impose additional restrictions that ensure the first order necessary conditions are also sufficient. The single-crossing condition, also known as the Spence-Mirrlees condition, is used to characterize the set of binding admissibility constraints. Combined with some restrictions on preferences, the first-order conditions become sufficient. See Salanie (2003) for details.

Assumption 8 The canonical dynamic private information economies typically do not converge. Immiseration is a property of the second-best allocation, prescribing that an increasingly large share of agents enjoy ever-decreasing consumption levels, even if aggregate consumption remains constant. (See Albanesi (2008) for a discussion.) As previously noted, imposing a lower bound on continuation utilities may render the constrained-efficient allocation stationary (Atkeson and Lucas, 1992).

[4] See Kehoe and Levine (1993) and Kocherlakota (1996).

3 Government under limited commitment

The government's policy choices may be constrained by a lack of commitment or by political economy considerations. These constraints often can be formulated as self-enforcement constraints on the government. We show how to fit them in our framework for both benevolent and self-interested policymakers.

3.1 Benevolent government

We illustrate this case with a simplified example from Reis (2006). The physical environment and the set of fiscal instruments are the same as in the simple Ramsey model described in section 1. The choice of policies is modeled as a game where both the government and private agents make *sequential* decisions in every period. In each period, both the government and private agents can default on outstanding debt obligations. The government decides how much to punish households who defaulted in the previous period, chooses taxes and transfers for the current period and decides whether to default on outstanding debt. Households then choose consumption, labor, capital and whether to default on debt. Finally, markets meet and clear.

The first admissibility constraint is the implementability constraint common to all Ramsey models and discussed in Section 1:

$$H_1(\psi, s_0) = \sum_{t=0}^{\infty} \beta^t h_1^0(x_t) \leq a_0$$

where

$$h_1^0(x_t) = u_t^c c_t + u_t^l l_t$$

and

$$a_1(s_0) = u_0^c \left\{ \left[\left(1 - \tau_0^k\right) F_0^k \right] k_0 + b_0 \right\}.$$

The implementability constraint only needs to be specified in the first period, thus constraints H_1 at nodes s^1 and later are trivially satisfied.

Let us now move to the limited commitment constraint. The value of default only depends on the state variable at the time of default and can be expressed as $V^{def}(k_{t-1})$. The value may be given by reversion to the worst sustainable equilibrium.[5] Alternatively default can be modeled as shutting down the government from the debt market, so it forces the government to run a balanced budget. Indeed it is also possible to include some arbitrary punishment mechanisms, like output loss or a shut down of private insurance markets.

The limited commitment constraint is then

$$U(x_t) \geq V^{def}(k_{t-1}).$$

Note the similarity with the self-enforcement constraints discussed in Section 2.1.

This formulation can be easily adapted to our framework by setting:

$$H_{2t}(\psi) = b_2(k_{t-1}) + \sum_{t=0}^{\infty} \beta^t h_1^0(x_t) \leq 0$$

where

$$b_2(k_{t-1}) = V^{def}(k_{t-1})$$

and

$$h_2^0(x_t) = -u(c_t, l_t).$$

[5]See Chari and Kehoe (1990) for a formal treatment of sustainable equilibria.

This constraint must be observed at all dates. The auxiliary variable is trivially set to zero everywhere.

3.2 Self-interested policymaker

We now turn to environments where the policymaker is not benevolent. As an example, we consider a simplified version of the economy in Acemoglu, Golosov and Tsyvinski (2008a, 2008b) with no aggregate or idiosyncratic risk. Thus $s^t = t$ trivially. To map this model into our framework we make use of the ex-ante heterogeneity, indexed by i in our notation. We let $i = 1$ index the representative household and $i = 2$ the rent-seeking policymaker. The latter values streams of transfers $\{T_t\}_{t\geq 0}$ according to the utility function:

$$U_2(x) = \sum_{t=0}^{\infty} \beta^t v(T_t).$$

Clearly we can equate the transfers T_t to the consumption allocation of the policymaker, c_{2t}.

The admissibility constraint arises as follows. At any date the ruler can capture a fraction of aggregate output by resorting to expropriation. Any admissible allocation must then satisfy:

$$\sum_{j\geq 0} \beta^j v(c_{2t}) \geq v(\kappa F(k_t, l_t)), \tag{5}$$

where the parameter κ, intended to capture the quality of political institutions in the model, represents the fraction of aggregate output that the ruler can extract.[6] This admissibility constraint has the same structure as the self-enforcement constraints considered in Section 2.1.

To map the admissibility constraint at date t set:

$$H_2(\psi, t) = \mathbf{b}_2(\psi, t) + \mathbf{h}_2^0(\psi, t)$$

with

$$\mathbf{b}_2(\psi, t) = v(\kappa F(k_t, l_t))$$

and

$$\mathbf{h}_2^0(\psi, t) = -\sum_{j\geq 0} \beta^j v(T_{t+j}).$$

Once again the auxiliary variable is trivially set to zero.

If the model calls for additional admissibility constraints on household allocations, we can simply stack them following the steps in the previous Sections.

The regularity conditions for this class of models are the same as those imposed for self-enforcement economies, discussed in Section 2.1 of this Appendix. Typically the regularity conditions are satisfied naturally in models with benevolent governments. For economies with a self-interested policymaker, it is usually necessary to impose some conditions on how the policymaker values transfers.

4 Principal-agent problems with investment

Our framework can also encompass some principal-agent problems with capital accumulation. We show how to map two models of firm dynamics, featuring moral hazard and limited commitment, respectively.

[6] Following expropriation, the ruler loses power and obtains no transfers.

4.1 Moral Hazard

We start with the principal-agent problem described in Clementi and Hopenhayn (2006). The first step to map their setting into our framework is to define the lender (the principal) and the entrepreneur (the agent) as ex-ante agents $i = 1$ and $i = 2$, respectively. The shock θ_t is technically an aggregate shock as it affects total resources available; although it remains private information to the entrepreneur and thus it is intuitively better understood as an idiosyncratic shock. The exogenous state is then $s^t = \theta^t$. As common in the contracting literature, Clementi and Hopenhayn (2006) frame the problem in terms of transfers across agents rather than consumption allocations. The transfer from the entrepreneur to the lender becomes the lender consumption, c_1, and the surplus becomes the entrepreneur consumption, c_2.[7] The resource constraint is:

$$c_1(s^t) + c_2(s^t) \leq F\left(k(s^{t-1}), s_t\right).$$

The incentive-compatibility constraint ensures the entrepreneur truthfully reports θ_t at all nodes. Misreporting allows the entrepreneur to forgo the transfer to the lender. In our notation, this is simply captured as the entrepreneur capturing all resources and consuming them. Thus, at node s^t the incentive-compatibility constraint is:

$$\sum_{j=0}^{\infty} \sum_{s^j \in S^j | s^t} \beta^j \pi(s^j | s^t) c_2(s^j) \geq F\left(k(s^{t-1}), s_t\right) + \sum_{j=1}^{\infty} \sum_{\tilde{s}^j \in S^j | \tilde{s}^t} \beta^j \pi(s^j | s^t) c_2(\tilde{s}^j)$$

where $\tilde{s}^t = \left\{\theta^{t-1}, \tilde{\theta}_t\right\}$ spans one-period misreports and their continuation histories.

The structure of the admissibility constraint is very similar to the case of private information in Section 2.2. The mapping to our framework thus follows similar steps. Let:

$$\begin{aligned}
\mathbf{b}(\psi, s^t) &= F\left(k(s^{t-1}), s_t\right), \\
\mathbf{h}^0(\psi, s^t) &= -\sum_{j=0}^{\infty} \sum_{s^j \in S^j | s^t} \beta^j \pi(s^j | s^t) c_2(s^j), \\
\mathbf{h}^0(\psi, \tilde{s}^t) &= \sum_{j=1}^{\infty} \sum_{\tilde{s}^j \in S^j | \tilde{s}^t} \beta^j \pi(s^j | s^t) c_2(\tilde{s}^j).
\end{aligned}$$

The auxiliary variable is trivially set to 0 at all nodes, and the admissibility constraint corresponds to:

$$H(\psi, s^t) = \mathbf{b}(\psi, s^t) + \mathbf{h}^0(\psi, s^t) + \mathbf{h}^0(\psi, \tilde{s}^t) \leq 0.$$

4.2 Limited enforcement

We now briefly discuss another principal-agent economy with a risk-neutral lender and entrepreneur, where the lending arrangement is subject to self-enforcement constraints. This example, due to Albuquerque and Hopenhayn (2004), is an application of the general model with self-enforcement constraints described in Section 2.1.

The setting is similar to Clemente and Hopenhayn (2006), but the admissibility constraint arises now from a self-enforcement condition and all information is public. We define the lender and entrepreneur as agents $i = 1$ and $i = 2$, respectively, and shocks as $s^t = z^t$. The resource constraint is identical to the previous case.

The entrepreneur can liquidate the project at any stage and obtain an outside option with value $O(k(s^{t-1}, s_t))$.[8] Thus, the self-enforcement constraint must ensure that the value of continuing the project for the entrepreneur is at

[7] Both agents are assumed to be risk-neutral, and thus the timing of the consumption is irrelevant.

[8] Albuquerque and Hopenhayn (2004) point out several possible specifications for function O.

least as high as the outside option:

$$\sum_{j=0}^{\infty} \sum_{s^j \in S^j | s^t} \pi(s^j|s^t)\beta^j c_2(s^j) \geq O(k(s^{t-1}), s_t).$$

This admissibility constraint is very similar to the case of self-enforcement constraint, described in Section 2.1. Let:

$$\begin{aligned} \mathbf{b}(\psi, s^t) &= O(k(s^{t-1}), s_t), \\ \mathbf{h}^0(\psi, s^t) &= -\sum_{j=0}^{\infty} \sum_{s^j \in S^j | s^t} \pi(s^j|s^t)\beta^j c_2(s^j), \end{aligned}$$

and set the auxiliary variable trivially to 0 at all nodes.

The regularity conditions for the principal-agent settings follow closely those for the respective economies, private information and self-enforcement constraints, discussed previously.

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