Sustainable Monetary Policy and Inflation Expectations

Roc Armenter*

Federal Reserve Bank of Philadelphia

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Abstract

The short-term nominal interest rate can anchor private-sector expectations into low inflation—more precisely, into the best equilibrium reputation can sustain. I introduce nominal asset markets in an infinite horizon version of the Barro-Gordon model and characterize the subset of sustainable policies compatible with any given asset price system at date $t = 0$. While there are usually many sustainable inflation paths associated with a given set of asset prices, the best sustainable inflation path is implemented if and only if the short-term nominal bond is priced at a certain discount rate at date $t = 0$.

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1 Introduction

Inflation in advanced countries has been low and stable for the last twenty years. Remarkably, central banks have been able to bring inflation under control while preserving their discretion at policymaking. Independence, transparency, and an emphasis on price stability are usually credited for the improved inflation record. Yet it remains important that we have a comprehensive understanding of how the current framework enables successful monetary policy, especially if additional institutional changes are being considered.

Most of the theoretical analysis of monetary policy frameworks builds upon the literature on commitment and discretion pioneered by Kydland and Prescott (1977) and Barro and Gordon (1983b). The initial premise is that there is a short-run benefit from unexpected inflation. If monetary policy is discretionary, the optimal policy is not compatible with rational expectations and the result instead is an inflationary bias. The institutional design is then modeled as modifying the policy objectives, either directly or through an inflation contract, in a way that reins in the ex-post incentives to inflate by the monetary authority.[1]

An old observation is that reputation can substitute for commitment: agents may forgo short-term benefits in order to secure cooperation over the long term. Reputation arises naturally in infinitely repeated games.[2] Barro and Gordon (1983a) show that low inflation can be sustained with trigger strategies in an infinitely repeated game between the monetary

[1] This approach started with the celebrated "conservative" central banker proposed by Rogoff (1985). Walsh (1995) first formalized inflation contracts. The subsequent literature is very large. Other strands of the literature emphasize learning or compare the performance of different Taylor rules. See Sargent (1999) and Clarida, Gali and Gertler (2000), respectively.

[2] This is not the only way to think about reputation, though. Barro (1986) and Backus and Driffill (1985), among others, show how uncertainty about the policymaker "type" allows for a different notion of reputation: the "good" policymaker strives to keep inflation low to distinguish himself from the "bad" policymaker. See Rogoff (1987) for a survey.

authority and the private sector. Formally, an inflation path is said to be sustainable (or enforceable) if it belongs to a sub-game perfect equilibrium. Much of the subsequent work has focused on the best sustainable policy.[3] Unfortunately infinitely repeated games have multiple equilibria, and some may be worse than the no-reputation equilibrium. Rogoff (1987) writes, "*Repeated game models replace a cooperation problem with a coordination problem.*" Torsten Persson and Guido Tabellini (1994) emphasize the "*problematic*" aspect of a "*serious multiple-equilibrium problem;*" while Thomas Sargent (1999) writes that "*the multitude of outcomes mutes the model empirically and undermines the intention of early researchers to use the rational expectations hypothesis to eliminate parameters describing expectations.*"[4]

In this paper I show that the short-term nominal interest rate can coordinate private-sector expectations into the best equilibrium reputation can sustain. Hence, all is required to solve the coordination problem is to set the discount price of a short-term nominal bond in a spot market—something central banks routinely do. Formally, I start with an infinitely repeated version of the economy in Barro and Gordon (1983a) and characterize all the sustainable equilibria. Then I introduce a date $t = 0$ spot market for nominal and real assets. By design, no action in the asset markets changes the set of sustainable equilibria—in other words, asset markets leave the cooperation problem intact. I show that the mapping between asset prices and sustainable policies is usually a correspondence, i.e., for a given asset price there are usually either many sustainable inflation paths compatible with an equilibrium. The exception, though, is of the utmost interest: there is a one-to-one mapping between the best sustainable policy and the highest discount price for the short-term nominal

[3] An incomplete list is Ireland (1997), Stokey (2002) and Atkeson and Kehoe (2005).

[4] The quotations are from Persson and Tabellini, eds (1994), p.9., and Sargent (1999), p. 38. The equilibrium multiplicity is sometimes regarded as a strength of the theory, as in Chari, Christiano and Eichenbaum (1998).

bond at date $t = 0$.

This result is made possible by three key properties of the best sustainable equilibrium. First, the inflation rate is a function of the exogenous state of the economy along the equilibrium path. Second, the inflation rate is the lowest of all sustainable inflation rates at all dates. Third, the only sustainable inflation path with the lowest inflation rate at date $t = 1$ is the best sustainable equilibrium. These properties are closely related to the bang-bang structure of the best sustainable equilibrium, documented in Abreu, Pearce and Stacchetti (1990).

Thus the coordination problem has moved from the very large space of possible inflation paths to a single spot market that central banks are very familiar with. There is no question that central banks have a good hold of the short-end of the nominal yield curve. For example, the Federal Reserve can set the rate in the federal funds market by manipulating the supply of reserves available to the banking sector. The federal funds rate is tightly link with short-term treasury bonds: through the available sample, the correlation between target federal funds rate and the 3-month Treasury bill rate is .99. Note that because the short-term nominal bond is traded at a discount price, in a spot market, there is no presumption that the monetary authority is committing to any action.

There are two important limitations to my analysis. First, I focus on how reputation can *lower* the inflation rate. Thus my analysis is entirely focused on the inflationary bias.[5] Similarly, I have to assume that the zero nominal interest rate bound is not binding in the best sustainable equilibrium. Otherwise, the short-term nominal interest fails to uniquely implement the best sustainable equilibrium, although it reduces the set of sequential equilibria to

[5] Arguably, advanced economies have kept inflation low for decades now though there is no consensus on how the inflationary bias has actually been solved. This paper suggests that central banks have simply been able to build their reputation once short-term nominal rates became the policy instrument of choice.

a desirable subset.

Finally, I include also a short discussion of inflation-targeting frameworks. To give inflation targeting the best shot at achieving coordination, I simply assume private-sector expectations are coordinated on the inflation target, however it is specified.[6] Then I ask under which conditions the inflation-target uniquely implements the best sustainable equilibrium and, if it fails to do so, what can be said about the associated set of sequential equilibria. I find it is possible to design the inflation target regime to guarantee that the best sustainable equilibrium is the unique sustainable equilibrium compatible with the target. Other target designs, commonly observed, fail to achieve unique coordination, but the associated set of equilibria retains desirable properties.

Section 2 presents the main result in the paper in a simple economy. The reader is referred to the Appendix for definitions, formal statements and proofs. Section 3 discusses the scope and limitations of the result. Section 4 discusses inflation targeting as a coordination device. Finally, Section 5 offers some concluding remarks.

2 Sustainable Monetary Policy

In this section I introduce the concept and key properties of sustainable policies in a simple economy. The Appendix contains the formal definitions, results and corresponding proofs in a more general environment.

[6] This is a strong assumption, as the inflation target would require coordination on future actions, unlike the present-period spot market implementation for the nominal interest rate.

2.1 A Simple Economy

I consider an infinitely repeated version of the economy proposed in Barro and Gordon (1983a). Monetary policy is the outcome of a game between two players, the monetary authority and the private sector. At every period $t = 1, 2...$ inflation expectations $\hat{\pi}_t$ are embodied in the private sector's actions, say, by setting nominal prices. The actual inflation rate is $\pi_t \geq \delta > 0$, and is directly set by the monetary authority. A Phillips curve determines aggregate output

$$y_t = y^* + \kappa \left(\pi_t - \hat{\pi}_t\right)$$

where $\kappa > 0$ and $y^* > 0$. Agents value output and inflation at date t according to

$$-\left(\bar{y} - y_t\right)^2 - \left(\pi_t - \delta\right)^2$$

where $\bar{y} > y^*$. The monetary authority is assumed to be benevolent: its objective function is the private-sector welfare. The mismatch between the equilibrium output y^* and the output "target" $\bar{y}$ reflects the presence of distortions.[7] The indirect utility function can be expressed as the negative of the loss function

$$l\left(\pi_t, \hat{\pi}_t\right) = \left(\bar{y} - y^* - \kappa\left(\pi_t - \hat{\pi}_t\right)\right)^2 + \left(\pi_t - \delta\right)^2.$$

The private sector's continuation welfare at any date $t \geq 1$ is

$$v_t = -\sum_{j=t}^{\infty} \delta^j l\left(\pi_j, \hat{\pi}_j\right)$$

[7] The formal results in the Appendix are derived using more general specifications for both the Phillips curve and the monetary authority's objective function. Foundations for this economy are discussed in Woodford (2003) and references. The use of a classical Phillips curve formulation is very convenient. The no-reputation equilibrium is basically an infinite sequence of static economies—hence the only intertemporal link possible is reputation. For an extension of infinitely repeated monetary policy games with endogenous state variables, see Chang (1998).

where $1 > \delta > 0$ is the intertemporal discount rate.

I start by solving for the optimal monetary policy. Rational expectations imply that there is no room for surprises, i.e., $\hat{\pi} = \pi$. Therefore the optimal monetary policy π^r solves

$$\min_{\pi \geq \delta} l(\pi, \pi)$$

at all dates. The optimal monetary policy is trivially given by the zero lower bound on the nominal interest rate, $\pi^r = \delta$. In more nuanced environments the optimal inflation rate may be positive and is usually a function of the state of the economy.

It is well-known that the monetary authority must be able to commit in order to implement the optimal policy in the one-period economy. Say the private sector expects the optimal monetary policy, $\hat{\pi} = \pi^r$, and the monetary authority has a chance to review the policy decision. Since $\hat{\pi}$ is already set, the monetary policy solves the *ex-post* problem:

$$\min_{\pi \geq \delta} l(\pi, \hat{\pi}).$$

The monetary authority's best response function is

$$\pi = \frac{\kappa(\bar{y} - y^*) + \delta}{1 + \kappa^2} + \frac{\kappa^2}{1 + \kappa^2}\hat{\pi}$$

and thus the monetary authority would choose to inflate over private-sector expectations if these were based on the optimal monetary policy:

$$\pi = \frac{\kappa(\bar{y} - y^*) + \delta}{1 + \kappa^2} + \frac{\kappa^2}{1 + \kappa^2}\pi^r > \pi^r.$$

Hence, the optimal monetary policy is not compatible with rational expectations and policy discretion.

What then is the rational expectations outcome in the one-period economy without commitment? We are looking for a fixed point in the best response function. That is, the monetary authority must find it optimal to validate the private-sector expectations,

$$\hat{\pi} = \arg\min_{\pi \geq \delta} l(\pi, \hat{\pi}).$$

It is straightforward to solve for the unique one-period Nash equilibrium π^n,

$$\pi^n = \delta + \kappa(\bar{y} - y^*).$$

Hence, the lack of commitment leads to the well-known inflation bias, $\pi^n > \pi^r$.

2.2 Sustainable Policy

In the infinite horizon economy, reputation can substitute for commitment and lower inflation. Say the private sector initially expects inflation to be below the one-period Nash equilibrium, $\hat{\pi} < \pi^n$. If monetary policy deviates at any date from expectations, $\pi_t \neq \hat{\pi}$, then the private sector subsequently coordinates on the one-period Nash equilibrium π^n. This strategy "punishes" the monetary authority whenever it exploits the Phillips curve. If the threat of higher inflation in future periods is large enough to deter the monetary authority, then it is possible to sustain lower inflation rates than π^n.

Whether any inflation path $\{\pi_t\}_{t=1}^{\infty}$ is *sustainable* in this manner is easy to check. At every date $t \geq 1$, the monetary authority can fulfill private-sector expectations and get the continuation value associated with $\{\pi_j\}_{j=t}^{\infty}$,

$$v_t = -\sum_{j=t}^{\infty} \delta^j l(\pi_j, \pi_j)$$

or it can choose some $\pi' \neq \pi_t$, knowing the one-period Nash equilibrium will follow forever after

$$-l(\pi', \pi_t) - \frac{\delta}{1-\delta} l(\pi^n, \pi^n).$$

The inflation path $\{\pi_t\}_{t=1}^{\infty}$ is a *sustainable plan* if the monetary authority chooses to fulfill private-sector expectations,

$$v_t \geq -l\left(\pi', \pi_t\right) - \frac{\delta}{1-\delta} l\left(\pi^n, \pi^n\right) \tag{1}$$

for all $\pi' \geq \delta$ at all dates $t \geq 1$.[8] In this simple economy, a sustainable inflation path can also be thought of as a *sustainable policy.*

The Folk theorem says that if the intertemporal discount rate is low enough, i.e., δ close enough to one, the optimal monetary policy should be sustainable. It is easy to check this. The continuation value associated with the infinite repetition of π^r is

$$v^r = -\frac{\left(\bar{y} - y^*\right)^2}{1-\delta}$$

while the best deviation achieves

$$-\frac{\left(\bar{y} - y^*\right)^2}{1+\kappa^2} - \frac{\delta}{1-\delta}\left(1+\kappa^2\right)\left(\bar{y} - y^*\right)^2$$

so the optimal monetary policy will be sustainable if

$$\frac{1}{1-\delta} \leq \frac{1}{1+\kappa^2} + \frac{\delta}{1-\delta}\left(1+\kappa^2\right)$$

or

$$\delta \geq \frac{1}{2+\kappa^2}.$$

Unfortunately, there are a lot of sustainable inflation paths. To start with, the infinite repetition of the one-period Nash equilibrium, $\pi_t = \pi^n$ for all $t \geq 1$, is always a sustainable plan. It can be shown that, if the optimal monetary policy is sustainable, then any constant

[8] The concept of a sustainable plan is broader: reversion to Nash equilibrium is not the only punishment possible and, more importantly, may not be the *worst* punishment possible. Hence, Condition (1) is sufficient but not necessary to establish an inflation path as sustainable. See the Appendix for a sufficient and necessary condition for a sustainable plan.

sequence in $[\pi^r, \pi^n]$ is sustainable too. Actually, the equilibrium multiplicity does not stop at deterministic sequences: it is also possible to sustain stochastic processes for inflation.[9]

I will be interested in the *best sustainable policy or plan* $\{\pi_t^*\}_{t=1}^{\infty}$, i.e., the sustainable inflation path that achieves the highest value at date $t = 1$, v_1. The optimal monetary policy, $\pi_t = \pi^r$ for all $t \geq 1$, may be a sustainable plan, in which case it trivially is the best sustainable plan. The cases of interest are those in which the optimal monetary policy is not sustainable, yet the best sustainable plan improves upon the one-period Nash equilibrium.

2.3 Key Properties of the Best Sustainable Plan

The results of this paper are built upon three key properties of the best sustainable plan. First, it is a constant sequence, $\pi_t^* = \pi^*$.[10] Second, it achieves the *lowest* inflation rate sustainable at every date. Third, as long as the lower bound on inflation is not binding, the only sustainable inflation path with $\pi_1 = \pi^*$ is the best sustainable plan.

In the Appendix I develop on the theory developed by Abreu et al. (1990) to formally prove these properties. Here I just sketch the logic behind them. Let $\{v_t^*\}_{t=1}^{\infty}$ be the sequence of continuation values associated with the best sustainable inflation path $\{\pi_t^*\}_{t=1}^{\infty}$. I can write the sustainability condition (1) at $t = 1$ as

$$v_1^* = -l\left(\pi_1^*, \pi_1^*\right) + \delta v_2^* \geq -l\left(\pi', \pi_1^*\right) - \frac{\delta}{1-\delta} l\left(\pi^n, \pi^n\right)$$

for all $\pi' \geq \delta$. Why is the best sustainable plan a constant sequence? If the continuation value were decreasing $v_1^* > v_2^*$, then one can just "postpone" the start of the best sustainable plan and do better. Note the alternative sequence $\{\pi_1^*, \pi_1^*, \pi_2^*, ...\}$ is sustainable since

$$-l\left(\pi_1^*, \pi_1^*\right) + \delta v_1^* > v_1^* \geq -l\left(\pi', \pi_1^*\right) - \frac{\delta}{1-\delta} l\left(\pi^n, \pi^n\right)$$

[9] See the Appendix for an example.

[10] For stochastic environments, the best sustainable plan is a function of the exogenous state of the economy. See the Appendix for details.

and it achieves strictly more welfare than $\{\pi_t^*\}_{t=1}^{\infty}$. A similar argument can be made if the continuation value were increasing $v_1^* < v_2^*$ by considering an alternative sequence $\{\pi_2^*, \pi_3^*, ...\}$ starting at date $t = 1$. Therefore the best sustainable plan achieves a constant value $v_t^* = v^*$ for all $t \geq 1$, which is only possible with a constant inflation rate $\pi_t^* = \pi^*$.

The second property—that the best sustainable plan achieves the lowest sustainable inflation rate at every period—follows easily now. Say a sustainable inflation path $\{\tilde{\pi}_t\}_{t=1}^{\infty}$ is such that, at some date t, $\tilde{\pi}_t < \pi^*$. The sustainability condition (1) at date t implies

$$-l(\tilde{\pi}_t, \tilde{\pi}_t) + \delta \tilde{v}_{t+1} \geq -l(\pi', \tilde{\pi}_t) - \frac{\delta}{1-\delta} l(\pi^n, \pi^n)$$

for all $\pi' \geq \delta$. Since $v^* \geq \tilde{v}_{t+1}$, the inflation path $\{\tilde{\pi}_t, \pi^*, \pi^* ...\}$ is also sustainable. Given that $l(\pi, \pi)$ is increasing in π, $\{\tilde{\pi}_t, \pi^*, \pi^* ...\}$ achieves

$$-l(\tilde{\pi}_t, \tilde{\pi}_t) + \delta v^* > v^*$$

which contradicts the definition of a best sustainable plan.

Finally, we are in place for the third property of the best sustainable plan: the only sustainable plan with $\pi_1 = \pi^*$, that is, the lowest inflation sustainable in the first period, is the best sustainable plan. I proceed to prove this result by contradiction. Assume a sequence $\{\pi^*, \{\tilde{\pi}_t\}_{t=2}^{\infty}\}$ is sustainable, with $\tilde{\pi}_t \neq \pi^*$ at some date $t \geq 2$. Condition (1) implies

$$-l(\pi^*, \pi^*) + \delta \tilde{v}_2 \geq -l(\pi', \pi^*) - \frac{\delta}{1-\delta} l(\pi^n, \pi^n).$$

Since I have already shown that the best sustainable plan is $\pi_t = \pi^*$, the continuation sequence $\{\tilde{\pi}_t\}_{t=2}^{\infty}$ must achieve strictly less value, $\tilde{v}_2 < v^* = -\frac{l(\pi^*, \pi^*)}{1-\delta}$. Hence

$$-\frac{l(\pi^*, \pi^*)}{1-\delta} > -l(\pi', \pi^*) - \frac{\delta}{1-\delta} l(\pi^n, \pi^n)$$

for all $\pi' \geq \delta$. The strict inequality above suggests that it may be possible to do better than π^*, that is, to lower inflation a bit and yet still satisfy Condition (1). This is exactly what I show next. Since the loss function l is continuous and convex in both arguments, the function

$$g(\varepsilon;\pi^*) \equiv -\frac{l\left(\pi^*-\varepsilon,\pi^*-\varepsilon\right)}{1-\delta}+\min_{\pi'\geq\delta} l\left(\pi',\pi^*-\varepsilon\right)$$

is also continuous around $\varepsilon=0$. It follows that for any $\kappa>0$ there exists $\varepsilon>0$ such that $|g(\varepsilon;\pi^*)-g(0;\pi^*)|<\kappa$. Therefore I can pick $\hat{\pi}=\pi^*-\varepsilon<\pi^*$ with $\varepsilon>0$ small enough such that

$$-\frac{l\left(\hat{\pi},\hat{\pi}\right)}{1-\delta}\geq -l\left(\pi',\hat{\pi}\right)-\frac{\delta}{1-\delta}l\left(\pi^n,\pi^n\right).$$

Hence the constant inflation path $\pi_t=\hat{\pi}$ is sustainable. Since $l\left(\hat{\pi},\hat{\pi}\right)<l\left(\pi^*,\pi^*\right)$, $\pi_t=\hat{\pi}$ for all $t\geq 1$ achieves higher welfare than the best sustainable plan $\hat{v}>v^*$, a contradiction.[11]

It is the third property that is key for the coordination, as it establishes that achieving a particular inflation rate in the first period is a sufficient and necessary condition for the best sustainable equilibrium. Next I will seek to exploit this property by linking the best sustainable equilibrium to a particular value of the short-term nominal interest rate.

2.4 Nominal Assets and Equilibrium Coordination

To make the point of this paper clear, I introduce asset markets in a way that neither the agent's payoff nor action sets are modified by asset trading. As a result, the set of sustainable inflation paths is left intact and asset prices are confined to a coordination role.

All asset trade happens at date $t=0$, one period before the game as described above starts. For simplicity, only a one-period nominal bond is traded here, sold at discount rate Q. There is no monetary phenomena at date $t=0$. All private-sector agents get an exogenous endowment $y_0<\bar{y}$.

[11] Readers familiar with Abreu et al. (1990) will recognize the bang-bang property of the best sequential equilibrium.

Private-sector agents optimally choose their bond holdings $\hat{B}$. Abstracting from borrowing constraints, the no-arbitrage condition implies

$$Q = \delta \frac{(\bar{y} - c_1)}{(\bar{y} - c_0)} \frac{P_0}{P_1}.$$

I assume that the monetary authority holdings B are exogenously dictated by a fiscal authority. Lump-sum transfers balance the monetary authority budget constraint and thus the resource constraint equates consumption with output. This renders the level of bond holdings irrelevant for the monetary authority decision problem at any date. It also does not affect the private-sector action set. As intended, the assumptions ensure that the set of sustainable plans is preserved.[12]

An asset market equilibrium combines a sustainable inflation path, the portfolio decision and the market clearing conditions. Since there are no surprises along the equilibrium path, the arbitrage condition simplifies to a simple Fischer equation,

$$Q = \delta \frac{\bar{y} - y^*}{\bar{y} - y_0} \frac{1}{\pi_1}. \tag{2}$$

An important assumption in the analysis is there is no sustainable inflation path that makes the zero nominal interest rate condition $Q \leq 1$ bind.

The arbitrage condition (2) is a one-to-one mapping between the nominal interest rate and inflation in the first period. However, the nominal interest rate does not seem to be informative with respect to the inflation rate in later periods. Moreover, not even the one-to-one relationship with period $t = 1$ inflation stands once the possibility of sustainable stochastic inflation paths is taken into account. There are usually many sustainable inflation paths compatible with the same nominal interest rate at date $t = 0$.

The key result of this paper is that, despite the limited scope of the arbitrage condition (2), there is a robust one-to-one relationship with the best sustainable plan. That is, if the

[12] Once again the formal definitions and proofs are in the Appendix.

asset market clears at price

$$Q^* = \delta \frac{\bar{y} - y^*}{\bar{y} - y_0} \frac{1}{\pi^*}$$

the only compatible inflation path is the best one, $\pi_t = \pi^*$ for all $t \geq 1$.

The properties of the best sustainable plan discussed in the previous subsection are now instrumental. First and foremost, it is sufficient to pin down the date $t = 1$ inflation rate at π^* to ensure that the best sustainable plan follows. Second, there is no room for stochastic inflation paths because π^* is the lowest sustainable inflation rate. Sitting at one extreme of the support for sustainable inflation rates, only degenerate lotteries are compatible with discount rate Q^*. It is also straightforward to see that Q^* is the highest discount price among the set of asset market equilibria.

The nominal interest rate also coordinates expectations into the best sustainable equilibrium in alternative market structures. In the Appendix I formally prove the result for a richer set of assets in a stochastic environment. Yet the result still hinges exclusively on the short-term nominal rate.[13]

3 Scope and Limitations

In this section I discuss the scope of my results as well as note some of the limitations of the analysis, concluding with some thoughts regarding the role of the nominal interest rate in providing some degree of coordination in general environments.[14]

[13] It is also possible, but cumbersome, to prove the result in a sequence-of-markets equilibrium.

[14] As mentioned earlier, I derive the results formally in the Appendix for a quite broad environment that includes shocks as well as general specifications for the Phillips curve and the monetary authority's loss function. The analysis of infinitely-repeated games, though, gets complicated quite easily–for example, by introducing endogenous state variables–so formal results for very general economies are difficult to come by.

There are two key conditions that frame the scope of my analysis. The first key condition is that, in absence of reputation, inflation is too *high*. In other words, the analysis is entirely focused on how reputation can solve the inflationary bias that arises from the time inconsistency problem. While such bias is pervasive in monetary economies, it is possible that inflation is too low due the monetary authority's lack of commitment.

The second key condition is that there is a strictly monotone relationship between the nominal interest rate and expected inflation. Obviously linear models satisfy this condition; non-linear models rarely have non-monotone comparative statics. However, economies in which the zero lower bound for the nominal rate binds in the short term violate the *strict* monotonicity required by the condition.

3.1 Inflationary bias

My analysis is focused on how reputation can lower the inflation rate: the no-reputation equilibrium inflation rate is assumed to be higher than the optimal inflation rate, with the best sustainable inflation rate being somewhere in between. This is certainly the typical case in models of inflationary bias, starting with the seminal work in Barro and Gordon (1983b) and Barro and Gordon (1983a). More generally, the inflationary bias arises whenever: (1) Unexpected inflation has an expansionary effect on real output, and (2) the monetary authority aims for more output than the natural level. I believe there is no need to discuss further (1), as it is ever-present in any model where monetary policy has a real effect. As for (2), there are strong structural foundations behind a loss function for the monetary authority with an output target above its natural level. As Benigno and Woodford (2005) show for New Keynesian models, if there are steady-state distortions in the underlying economy, a loss function based on a linear-quadratic approximation will have the required property.[15]

As advanced economies have managed to keep inflation low for decades now, researchers routinely assume away the inflation bias by setting the monetary authority's target to the natural level, so inflation is, on average, optimal. This does not mean that there are no time-inconsistency issues: in New Keynesian models, the no-reputation equilibrium will typically feature excessive inflation volatility relative to the optimal policy, in what is known as *stabilization* bias.[16] As shown by Kurozumi (2008), reputation can help ameliorate the stabilization bias and even implement the optimal policy.

Unfortunately, the nominal interest rate may not be able to coordinate expectations into the best sustainable policy if the stabilization bias is large relative to the inflationary bias. More precisely, if the optimal inflation rate is higher than the inflation rate in the no-reputation equilibrium, then setting the nominal interest rate at the level implied by the best sustainable inflation does not rule out sustainable plans with lotteries. These equilibria will have the same *expected* inflation as the best sustainable plan, but inflation realizations can be spread over the support of all sustainable plans and thus are sub-optimal. In contrast, when the best sustainable inflation is the lowest inflation rate sustainable, that is, sits at the boundary of the support of all sustainable plans, the nominal interest rate will pin down the inflation rate without room for lotteries. Hence whether my results apply in an economies that have both inflationary and stabilization bias depends on the underlying parameters governing the strength of each bias.

[15]New Keynesian models specify forward-looking terms in the Phillips curve. See Woodford (2003) for a comprehensive review.

[16]See McCallum (1997) and Clarida, Gali and Gertler (1999) for early discussion of the phenomena.

3.2 Nominal interest rate and expected inflation

The second key condition requires that the nominal interest rate and expected inflation have an one-to-one relationship. This condition is satisfied in models where monetary policy does not effect real output along the equilibrium path.[17] In such models, the real interest rate is constant and the Fischer equation (2) delivers the monotone relationship that we are looking for.[18]

There are models, most notably those embedding a New Keynesian Phillips curve, where monetary policy moves the real interest rate along the equilibrium path. It is thus possible that the relationship between the nominal interest rate and expected inflation is non-monotone. That said, these models are usually log-linearized, thus obviously satisfying the required condition. I should also note that the condition can be weakened to hold only for the best sustainable plan.

Similarly, the relationship between expected inflation and the nominal interest rate usually does not depend on the asset-trading arrangement: whether one assumes markets are complete or incomplete, or bonds are exclusively in nominal terms, it is irrelevant for the Fischer equation. However, a more realistic modeling of the asset market is bound to modify the set of sustainable plans. Nominal bond holdings are likely to shape the ex-post incentives of the monetary authority—see, for example, Calvo (1978). Moreover, the short-term interest rate is often not only a nominal anchor but also the operative instrument of monetary policy. I abstracted from these possibilities in order to focus on the coordination role.

There is, however, a relevant situation for which the conditions fails: If the best sustainable plan calls for the nominal interest rate to be at the zero lower bound at date $t = 1$, then

[17] The one-to-one relationship does not need to hold off the equilibrium path, that is, for plans that are not sustainable.

[18] More generally, the real interest would be a function of the exogenous state of the economy.

it is not possible to coordinate uniquely on the best sustainable plan. A sustainable plan may feature an inflation rate being equal to that under the best sustainable plan in date $t = 1$—or, more broadly, for as long as the zero lower bound binds—yet depart from the best sustainable plan at a later date. In New Keynesian models, the so-called forward guidance channel would imply very different outcomes across equilibria, as inflation expectations after the nominal rate departs the zero lower bound have a large impact on the present-period real interest rate.

3.3 Partial coordination

Even when the above conditions do not hold, the nominal interest rate still provides some degree of coordination for private-sector expectations. To see this, it is necessary to entertain the possibility of stochastic sustainable plans. Returning to the simplified model of the previous section, let $\Pi = [\pi_{lb}, \pi_{ub}]$ be the support of sustainable inflation rates for period $t = 1$. It is well-known that if any inflation rate in the support Π is sustainable, then any probability measure μ over Π for period $t = 1$ is also a sustainable plan.[19] For any such sustainable plan μ, I can then write the Fischer equation (2) as

$$R^{-1} = \delta \int_{\pi_{lb}}^{\pi_{ub}} (\pi)^{-1} \mu(d\pi), \tag{3}$$

where for simplicity I have assumed $y_1 = y_0$ and dropped time subscripts. The set of sustainable plans μ that are compatible with a particular nominal interest rate R is simply the set of probability measures μ over Π that satisfy (3). It is only possible to implement uniquely the *boundaries* of Π, say, $\pi = \pi_{lb}$, by setting $R = \pi_{lb}\delta^{-1}$. When π_{lb} happens to be also the best sustainable plan we obtain my main result.

What happens if the nominal interest rate is set to $R > \pi_{lb}\delta^{-1}$, perhaps because higher

[19] See Appendix for a detailed treatment of stochastic sustainable plans.

inflation is preferable or there is some disconnect between the policy rate and the short-term nominal rate? One possible sustainable equilibrium compatible with R would be to have inflation $\pi_1 = R\delta$ with probability one. There are stochastic sustainable plans that are compatible with R as well, but the probability measure must respect equation (3). This is a sizable constraint over the set of sustainable plans—particularly if $R\delta$ is near the boundaries of the support Π and the probability measure μ cannot have a lot of mass on inflation rates very different from $R\delta$ and satisfy (3). It is quite simple to show that, for any $\hat{\pi} \in \Pi$ and given R,

$$\Pr(\pi > \hat{\pi}) \leq \left(1 - \frac{\pi_{lb}}{R\delta}\right)\left(1 - \frac{\pi_{lb}}{\hat{\pi}}\right)^{-1}.$$

A similar bound can be obtained using the upper bound of the support Π. I should note that the above bound only applies to the first-period inflation rate. Perhaps not surprisingly, the bounds for the inflation rate at longer horizons are less informative, that is, the short-term nominal rate at date $t = 0$ becomes compatible with a broader set of sustainable inflation rates at later dates. However, these bounds are quite tight if the nominal rate is set close to $\pi_{lb}\delta^{-1}$ as the first term in the probability bound is close to zero. Indeed, the underlying correspondence between the short-term nominal interest rate and the set of sustainable equilibria is continuous, implying that, in general, small deviations in the short-term nominal interest rate would produce sustainable equilibria that are, welfare wise, very close to the best sustainable equilibrium.

An alternative approach would be to compute the *worst* sustainable plan compatible with a given nominal interest rate R. This is particularly useful for analyzing the case of a binding zero lower bound. Borrowing loosely from the specification in Eggertsson and Woodford (2003), assume the economy is under a negative, persistent real interest rate shock that leads the best sustainable policy to the zero lower bound. With a constant probability, the real interest rate returns to steady state and the best sustainable policy exits the zero lower bound. Note that the worst sustainable plan still requires some reputation, i.e., a continuation value, to enforce date $t = 1$ inflation compatible with the nominal rate being at the zero lower bound. Since the continuation value must be delivered along the equilibrium path, there are some restrictions. For example, the worst sustainable plan may feature departing the zero lower bound early—but not *too* early or the low inflation at date $t = 1$ would have not been sustainable. Unfortunately it is difficult to give a precise evaluation of how far the worst sustainable equilibrium (compatible with the initial interest rate) would depart from the best sustainable equilibrium.

4 Inflation targeting

Inflation targeting has become the monetary policy framework of choice in several countries since New Zealand pioneered its use in 1990. The details vary from country to country, but at broad strokes inflation targeting can be said to consist of a range and/or an average for the growth rate of a price level of choice, typically a consumer price index, to be achieved over a medium-term horizon, e.g., two years. Inflation-targeting central banks still use some short-term nominal interest rate as instrument and part of the policy communication.

In this Section I ask whether an inflation target is an effective coordination device. I assume private-sector expectations are coordinated on the inflation target, however it is specified. Then I check whether this is sufficient to uniquely implement the best sustainable equilibrium and, if it is not, what can be said of the associated set of sustainable equilibria. I do not specify how the monetary authority achieves coordination on the target: this is not trivial since inflation targets require coordination on future date outcomes, rather than in a present-period spot market as in the case of the nominal interest rate.

Formally, I define an inflation-target regime as (1) a subset $\bar{\Pi} \subseteq \{\pi \geq \delta\}$, and (2) a time horizon $\bar{T} \subseteq \{0, 1, \dots\}$.[20] This encompasses "point" targets, a range or a ceiling for inflation... at different horizons. An inflation plan complies with the target if $\pi_t \in \bar{\Pi}$ for all dates $t \in \bar{T}$. I then ask for which inflation-target frameworks the only sustainable plan that complies with the target is the best sustainable plan and, if there are others, what can be said about them.[21]

Let us start with a point value for inflation, at some horizon T, as the target, that is, $\bar{\Pi} = \hat{\pi}$. On the heels of my results it should not be surprising that it is possible to pick $\hat{\pi}$ such that the best sustainable plan π^* is uniquely determined if $T = \{1\}$. Setting $\hat{\pi} = \pi_1^*$ achieves coordination on the best sustainable equilibrium: since π_1^* is the lower bound on the support for any sustainable inflation rate, there is no room for stochastic sustainable equilibria. The argument sketched here carries nicely to stochastic economies and targets defined over the *average* inflation rate.

However, the condition that $T = \{1\}$ is important. What happens if the target is set for later horizons? By setting $\hat{\pi} = \pi_t^*$ for $t > 1$ all compatible sustainable plans will implement the best continuation equilibrium from date t onwards, but there is no guarantee that the best sustainable equilibrium is played. For example, a possible sustainable equilibrium would be to play the one-period Nash equilibrium until date $t-1$, and then revert to the best continuation equilibrium at date t. If the target is specified at $\hat{\pi} = \pi_t^*$ for several horizons $t \in T$, only the shortest horizon date is relevant.

Perhaps not surprisingly, the benefits of a point target diminish once you consider a stochastic economy. In any case, many inflation-targeting frameworks specify a range for inflation, rather than target a point. The range is unconditional, and again over some horizon

[20] This can be easily generalized further to stochastic economies and a richer specification for the inflation target, like, for example, a range for the average inflation rate.

[21] It may be possible that no sustainable plan complies with the target. See Kurozumi (2012) for the limits of inflation-targeting designs based on reputation.

T. Consider thus $\hat{\Pi} = [\pi_{lb}, \pi_{ub}]$. In a deterministic setting, the range can work if the upper bound is set to be equal to the best sustainable inflation rate, π^*. However, this does not work for stochastic environments. Let $s \in S$ be the exogenous state of the economy, evolving according to some Markov process.[22] The obvious choice for π_{ub} would be $\max_{s\in S} \pi^*(s)$, that is, the maximum inflation rate attained under the best sustainable equilibrium. However, it is not possible to uniquely implement the best sustainable equilibrium. If $T = 1$, the best continuation value is only guaranteed if $s_1 = \arg\max_{s\in S} \pi^*(s)$, that is, if the state realization at date $t = 1$ is such that the upper bound on the range is binding. The same problem arises for longer horizons as well. Indeed, even if the range is imposed at all dates, the best sustainable equilibrium is not the unique sustainable equilibrium compatible. For example, inflation may be at the one-period Nash equilibrium until the exogenous state is $\arg\max_{s\in S} \pi^*(s)$. On the positive side, all sustainable equilibrium will achieve the best continuation value with probability one if the range is imposed at all dates.

To recap, it is possible to uniquely implement the best sustainable equilibrium by targeting the average inflation rate in the short term. Other inflation-targeting frameworks do not uniquely pin down the best sustainable equilibrium, but they still guarantee that only a subset of sustainable equilibrium, with quite desirable properties, is implemented. It is worth insisting, though, that this analysis simply assumes coordination on the target, without detailing how it is achieved. That said, it is quite intriguing that an easily communicated target can bring a fair degree of coordination on the much larger object that the set of sustainable equilibria is.

[22] See Appendix for a proper formulation.

5 Concluding remarks

Advanced economies have kept inflation low for decades now yet there is no consensus on how the inflationary bias has actually been solved. At its broadest interpretation, this paper suggests that central banks may have simply been able to build on their reputation by making short-term nominal rates their choice of policy instrument. I show that manipulating the discount price of the short-term nominal bond in a spot market—something central banks around the world seem to have no problem doing—can effectively coordinate private-sector expectations on the best sustainable inflation path.

Perhaps the biggest gap between theory and practice in my analysis is the construct of a date $t = 0$, when no equilibrium is yet selected and the monetary authority has the ability to coordinate beliefs. Moreover, it is important that coordination is a once-and-for-all event. That is, agents' initial beliefs regarding the equilibrium to be played are taken to be heterogeneous, that is to say, uncoordinated. However, once coordination is achieved, the central bank no longer can reset private expectations at will. This is crucial for the theory: reputation is only as valuable as it is costly to lose it. If the monetary authority could coordinate expectations into the best sustainable policy at any time, it would be possible to get out of any punishment phase and no equilibrium other than the one-period Nash equilibrium could be sustained. In other words, if the central bank could build up reputation at will, it would render reputation useless in the first place.

The natural next step would be to embed the analysis in a learning model with dispersed beliefs over the set of sustainable equilibria. In such model, agents would gradually update their posterior beliefs into the sustainable equilibrium that are compatible with the nominal interest rate set by the monetary authority. Instead of the date $t = 0$ construct, one would start with uninformative priors over the set of equilibria. As the agents' posterior converge to the best sustainable plan, that is, once coordination is achieved, the monetary authority would face a much harder time shifting these beliefs again—and thus preserving the role of reputation.

There remain two substantial challenges for combining a learning model and a theory of reputation. First, we do not have tractable models of coordination. An exception is the fictitious play model of Fudenberg and Kreps (1993) and Fudenberg and Takahashi (2008). There is also some work documenting how second-price sealed-bid auctions achieve coordination on Pareto superior equilibria.[23] However, in both cases, all learning is completed in the first period. Second, sustainable equilibria rely heavily on out-of-the-equilibrium path beliefs, in the form of trigger strategies. A learning model is bound to impact the cooperation problem, as the monetary authority may be able to carry on with a deviation for longer than under the rational expectations assumption.

[23]See Huyck, Battalio and Beil (1993), Crawford and Boseta (1998), and Janssen (2006). Interestingly, open market operations are an example of such auction design.

References

Abreu, Dilip, "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, March 1988, *56* (2), 383–396.

——, **David Pearce, and Ennio Stacchetti**, "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring," *Econometrica*, September 1990, *58* (5), 1041–1063.

Atkeson, Andrew and Patrick J. Kehoe, "The Advantage of Transparency in Monetary Policy Instruments," 2005. Working Paper.

Backus, David and John Driffill, "Inflation and Reputation," *The American Economic Review*, 1985, *75* (3), 530–538.

Barro, Robert J., "Reputation in a Model of Monetary Policy with Incomplete Information," *Journal of Monetary Economics*, 1986, *17*, 1–20.

—— **and David B. Gordon**, "Rules, Discretion and Reputation in a Model of Monetary Policy," *Journal of Monetary Economics*, 1983, *12*, 101–120.

—— **and D.B. Gordon**, "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, 1983, *91* (4), 589–610.

Benigno, Pierpaolo and Michael Woodford, "Inflation stabilization and welfare: The case of a distorted steady state," *Journal of the European Economic Association*, 2005, *3* (6), 1185–1236.

Calvo, Guillermo A., "On the Time Consistency of Optimal Policy in a Monetary Economy," *Econometrica*, 1978, *46*, 1411–1428.

Chang, Roberto, "Credible Monetary Policy in an Infinite Horizon Model: Recursive Approaches," *Journal of Economic Theory*, 1998, *81*, 431–461.

Chari, V.V. and Patrick J. Kehoe, "Sustainable Plans," *The Journal of Political Economy*, August 1990, *98* (4), 783–802.

____ , Lawrence J. Christiano, and Martin Eichenbaum, "Expectation Traps and Discretion," *Journal of Economic Theory*, August 1998, *81* (2), 462–492.

Clarida, Richard, Jordi Gali, and Mark Gertler, "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, December 1999, *37* (4), 1661–1707.

____ , ____ , and ____ , "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 2000, *115* (1), 147–180.

Crawford, Vincent and Bruno Boseta, "What Price Coordination? The Efficiency-Enhancing Effect of Auctioning the Right to Play," *The American Economic Review*, March 1998, *88* (1), 198–225.

Cronshaw, Mark B. and David G. Luenberger, "Strongly Symmetric Subgame Perfect Equilibria in Infinitely Repeated Games with Perfect Monitoring and Discounting," *Games and Economic Behavior*, 1994, *6*, 220–237.

Eggertsson, Gauti B and M. Woodford, "Zero bound on interest rates and optimal monetary policy," *Brookings Papers on Economic Activity*, 2003, *2003* (1), 139–233.

Fudenberg, Drew and David M. Kreps, "Learning Mixed Equilibria," *Games and Economic Behavior*, July 1993, *5* (3), 320–367.

—— **and Satoru Takahashi**, "Heterogeneous Beliefs and Local Information in Stochastic Fictitious Play," 2008. Working paper.

Huyck, John B. Van, Raymond C. Battalio, and Richard O. Beil, "Asset Markets as an Equilibrium Selection Mechanism: Coordination Failure, Game Form Auctions, and Tacit Communication," *Games and Economic Behavior*, 1993, *5*, 485–504.

Ireland, Peter, "Sustainable Monetary Policies," *Journal of Economic Dynamics and Control*, 1997, *22*, 87–108.

Janssen, Maarten C.W., "Auctions as coordination devices," *European Economic Review*, 2006, *50*, 517–532.

Kurozumi, Takushi, "Optimal sustainable monetary policy," *Journal of Monetary Economics*, 2008, *55* (7), 1277–1289.

—— , "Sustainability, flexibility, and inflation targeting," *Economic Letters*, 2012, *114*, 80–82.

Kydland, Finn and Edward C. Prescott, "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 1977, *85* (3), 473–491.

Ljungqvist, Lars and Thomas J. Sargent, *Recursive Macroeconomic Theory*, London, England: The MIT Press, 2000.

McCallum, Bennett T, "Crucial issues concerning central bank independence," *Journal of Monetary Economics*, 1997, *39* (1), 99–112.

Persson, Torsten and Guido Tabellini, eds, *Monetary and Fiscal Policy*, Cambridge, Massachusetts: The MIT Press, 1994.

Rogoff, Kenneth, "The Optimal Degree of Commitment to an Intermediate Monetary Target," *Quarterly Journal of Economics*, 1985, *100*, 1169–1189.

——, "Reputational Constraints on Monetary Policy," *Carnegie-Rochester Conference Series on Public Policy*, 1987, *26*, 141–182.

Sargent, Thomas J., *The Conquest of American Inflation*, Princeton University Press, 1999.

Stokey, Nancy, "'Rules versus Discretion' After Twenty-Five Years," 2002. NBER Macroeconomics Annual.

Walsh, Carl E., "Optimal Contracts for Central Bankers," *American Economic Review*, March 1995, *85* (1), 150–167.

Woodford, Michael, *Interest and Prices*, Princeton: Princeton University Press, 2003.

A Appendix

A.1 Environment and Sustainable-Equilibrium Definition

This is an infinite horizon economy $t = 1, 2, ...$ Monetary policy is characterized as the outcome of a repeated game between the monetary authority and a continuum I of agents, which are denoted as a whole as the private sector.

At the beginning of each period $t \geq 1$, the exogenous state s_t becomes common knowledge. I assume that the exogenous state $s_t \in S$ is governed by a stochastic first-order Markov probability process μ.

After the exogenous state is observed, private-sector agents choose some variable, $z_{it} \in Z$. For example, some firms may set nominal prices in advance or unions may agree on nominal wages for the period. Let $z_t = \int z_{it} di$ be the average of all agents decisions. I will assume that the private-sector agent's and monetary authority's payoffs depend on actions $\{z_{it}\}_I$ only through its average z_t. This implies that a private-sector competitive equilibrium can be thought of as the average action z_t^* solving the optimality conditions $z_t^* = \zeta\left(z_t^*, \hat{\pi}_t\right)$, where $\hat{\pi}_t$ is the private-sector inflation expectation. Furthermore, I assume that, given an inflation expectation $\hat{\pi}_t$, the private-sector competitive equilibrium is unique. Hence, the private-sector competitive equilibrium can be summarized by $\hat{\pi}_t$, which is treated as the "action" of the private sector.

Finally, the monetary authority sets the actual inflation rate π_t taking as given the private sector's inflation expectations $\hat{\pi}_t$. Inflation is bounded below by the zero-lower bound in the nominal interest rate, $\pi_t \in \Pi \equiv \{\pi \geq \delta\}$ where $\delta > 0$.

The economy considered is very simple. Output and inflation are linked by a classic Phillips curve,

$$y_t = y\left(\pi_t - \hat{\pi}_t; s_t\right)$$

where $y(\pi_t - \hat{\pi}; s_t)$ is strictly positive, increasing in the first argument, and bounded above. Let $y^*(s_t) \equiv y(0, s_t)$. Output is transformed one-to-one into a non-storable consumption good. The aggregate resource constraint is then $c_t \leq y_t$.

The household period payoff is given by $u(y_t) - g(\pi_t)$, where u is a strictly increasing, concave, and twice differentiable function; and g is a strictly increasing and twice differentiable equation. Function u is assumed to be bounded below and g bounded above. Finally, I assume that the monetary authority is benevolent so its period payoff is given by $u(y_t) - g(\pi_t)$ as well.[24]

I briefly introduce the time-inconsistency problem in a one-period version of this economy. First, I will define an one-period Nash equilibrium—the discretionary equilibrium in the language of Barro and Gordon (1983b). I will also characterize the one-period optimal monetary policy.

Definition 1 *A one-period Nash equilibrium at state s is $\pi \geq \delta$ such that*

$$u(y^*(s)) - g(\pi) \geq u(y(\pi' - \pi; s)) - g(\pi')$$

for all $\pi' \geq \delta$.

I will assume that there exists a one-period Nash equilibrium for each state $s \in S$.

Definition 2 *The optimal monetary policy at state s is $\pi \geq \delta$ such that*

$$u(y^*(s)) - g(\pi) \geq u(y^*(s)) - g(\pi')$$

for all $\pi' \geq \delta$.

[24] Foundations for this economy are discussed in Woodford (2003) and references.

The optimal monetary policy is trivially given by $\pi = \delta$ for all $s \in S$.[25] The time-inconsistency problem arises when the two equilibrium concepts are not equivalent, i.e., the optimal monetary policy is not the unique one-period Nash equilibrium. If this is the case, some commitment technology is required to implement the optimal monetary policy as the unique rational expectations equilibrium.

The set of Nash equilibrium is usually enlarged in the infinite horizon economy. In particular, history-dependent equilibria arise, and some of the equilibria feature trigger strategies that resemble reputation.

I start by introducing the notation for histories.[26] Let $\pi^t = \{\pi_1, \pi_2 ..., \pi_t\} \in \Pi^t$ denote the history of the monetary authority's actions up to date $t \geq 1$. In a similar fashion define $\hat{\pi}^t \in \Pi^t$ and $s^t \in S^t$ for $t \geq 1$. To denote the set of continuation histories at s^t, I use $S^j | s^t$, $j \geq t$. The payoff-relevant node is $h^t = \{\pi^t, \hat{\pi}^t, s^t\} \in H^t \equiv \Pi^{2t} \times S^t$ for $t \geq 1$, with $h^0 = \{\varnothing\}$. Node h^t is also the end-of-the-period history.

A consumption plan $c = \{c(h^t) : h^t \in H^t, t \geq 1\}$ specifies a level of consumption at every node $h^t \in H^t$, $t \geq 1$. Let F be a probability measure over all game nodes H^t, $t \geq 1$. The private-sector welfare at any h^t, $t \geq 1$, is defined over c given F

$$v\left(c; h^t, F\right) = \sum_{j=0}^{\infty} \sum_{s^{t+j} \in S^{t+j}} \delta^j \mu\left(s^{t+j} | s^t\right) \int_{H^{t+j}} \left(u\left(c\left(h^{t+j}\right)\right) - g\left(\pi_{t+j}\right)\right) F\left(dh^{t+j} | h^t, s^{t+j}\right) \quad (4)$$

where $0 < \delta < 1$ is the intertemporal discount rate. I stretch the definition of v to ex-ante

[25] This "boring" optimal monetary policy ensures that the analysis is focused on how reputation can achieve *lower* average inflation.

[26] I assume that strategies can only be conditional on public histories and not on the history of private actions $\{z_{it}\}_{t=1}^{\infty}$.

welfare

$$
\begin{aligned}
v(c; s, F) &= \int_H v\left(c; h^1, F\right) F\left(dh^1|s\right), && (5) \\
v(c; F) &= \sum_{s \in S} \mu(s|s_0)\, v(c; s, F). && (6)
\end{aligned}
$$

Finally, the resource constraint is given by

$$
c\left(h^t\right) = y\left(\pi_t - \hat{\pi}_t; s_t\right) \tag{7}
$$

for all $h^t \in H^t$, $t \geq 1$.[27]

Now I describe the strategy space. For the private sector, the relevant decision node is (h^{t-1}, s_t) as the present state of the economy is known in addition to the history of past actions.[28] The period strategy is denoted by $\hat{\sigma}(h^{t-1}, s_t) \in \Delta(\Pi)$. The strategy space contains mixed strategies in order to allow equilibria where private-sector expectations are driven by a sunspot variable. The private-sector strategy is then

$$
\hat{\sigma} = \left\{\hat{\sigma}\left(h^{t-1}, s_t\right) : \left(h^{t-1}, s_t\right) \in H^{t-1} \times S^t, t \geq 1\right\}.
$$

The monetary authority is aware of the history of actions, the state of the economy, and the realization of the private-sector strategy. The relevant node for the monetary authority is then $x^t \equiv (h^{t-1}, s_t, \hat{\pi}_t) \in X^t \equiv H^{t-1} \times S^t \times \Pi$ and its period strategy is $\sigma(x^t) \in \Pi$. Let

$$
\sigma = \left\{\sigma\left(x^t\right) : x^t \in X^t, t \geq 1\right\}
$$

denote the monetary authority strategy.

The chosen equilibrium concept is the sequential equilibrium.

[27] The resource constraint is stated with strict equality sign. This turns out to be important since it rules out punishment strategies based on disposal of resources—what are known as "burning money" strategies.

[28] It may sound awkward to talk of the private-sector *strategy* given that the private-sector is not a rational player but a continuum of small agents. The distinction will be contained in the equilibrium definition; the term strategy is just convenient.

Definition 3 *A sequential equilibrium (SE) is a strategy profile* $\{\hat{\sigma}, \sigma\}$*, a probability distribution* F *and a consumption plan* c *such that:*

1. *For all* $x^t \in X^t, t \geq 1$

$$v\left(c; \left\{x^t, \sigma\left(x^t\right)\right\}, F\right) \geq v\left(c; \left\{x^t, \pi'\right\}, F\right)$$

   for all $\pi' \geq \delta$*;*

2. *For all* $(h^{t-1}, s_t) \in H^{t-1} \times S, t \geq 1$*,* $\hat{\sigma}$ *is such that*

$$\hat{\pi}_t = \sigma\left(x^t\right)$$

   almost everywhere in $H^t|X^t$*;*

3. *For all* $h^t \in H^t, t \geq 1$*, the resource constraint (7) holds;*

4. *Probability measure* F *is generated from* $\{\hat{\sigma}, \sigma\}$*.*

The key elements of the sequential-equilibrium definition are the subgame perfection requirement and rational expectations. The former ensures that the monetary authority's decision maximizes private-sector welfare at every decision node—this is Condition 1 in the definition. Condition 2 imposes rational expectations: it implies that the private sector has the correct beliefs with respect to the actual inflation rate along the equilibrium path. Note that Conditions 1 and 2 are very different. The private-sector "player," actually a continuum of atomistic agents, does not behave strategically.

It is straightforward to show that the infinitely repeated version of a one-period Nash equilibrium is a SE. Usually there will be many SE with distinct welfare properties. The SEs that deliver the highest welfare possible are of obvious interest.

Definition 4 *A best sequential equilibrium (BSE) is a sequential equilibrium* $\{\sigma, \hat{\sigma}, F, c\}$ *such that*

$$v\left(c; F\right) \geq v\left(c'; F'\right)$$

for all sequential equilibria $\{\sigma', \hat{\sigma}', F', c'\}$.

Before proceeding to the discussion of the equilibrium properties, I define a monetary policy plan as a probability measure M over $\Pi^t \times S^t$. Clearly, for a given SE $\{\sigma, \hat{\sigma}, F, c\}$, the probability measure F defines a monetary policy plan. Following Chari and Kehoe (1990), a monetary policy plan will be sustainable if there exists a SE generating the policy plan.

Definition 5 *A probability measure* M *over* $\Pi^t \times S^t$ *is a sustainable monetary policy plan if there exists a sequential equilibrium* $\{\sigma, \hat{\sigma}, F, c\}$ *such that* M *is generated by* F.

I will often use sustainable plan for short.

A.2 Sustainable-Equilibrium Properties

A key object in the analysis of SE is the equilibrium value set, i.e., the set of private-sector welfare associated with a SE. It will prove instrumental to characterizing the BSE. For convenience, I first define the equilibrium value set conditional on the state s,

$$V\left(s\right) \equiv \left\{v\left(c; s, F\right) \mid \{\sigma, \sigma', F, c\} \text{ is a SE}\right\}$$

and then the ex-ante welfare set

$$V \equiv \left\{v\left(c; F\right) \mid \{\sigma, \sigma', F, c\} \text{ is a SE}\right\}.$$

As mentioned earlier, the infinitely repeated version of a one-period Nash equilibrium is a SE. Hence $V\left(s\right)$ is not empty for all $s \in S$.

The remainder of this section builds on the seminal work of Abreu et al. (1990) to characterize the set $V(s)$ and then the BSE. The private-sector welfare can be written recursively. From (4), the end of period welfare can be decomposed in the period payoff and the continuation value,

$$v\left(c;h^t,F\right)=u\left(c\left(h^t\right)\right)-g\left(\pi_t\right)+\delta\sum_{s^{t+1}}\mu\left(s^{t+1}|s^t\right)\int_{H^{t+1}}v\left(c;h^{t+1},F\right)F\left(dh^{t+1}|h^t\right).$$

The key observation is that every subgame in a SE constitutes a SE as well. This implies that, at any node of the game, the continuation payoffs must belong to $V(s)$ as well, i.e.,

$$\int_{H^t}v\left(c;h^t,F\right)F\left(dh^t|h^{t-1},s_t\right)\in V\left(s_t\right)$$

for all $\left(h^{t-1},s_t\right)\in H^{t-1}\times S$, $t\geq 1$.

The next step is to define a sustainable action $\pi\in\Pi$. Sustainable actions are the building blocks of a SE. Loosely speaking, for an action π_t to be sustainable, there should be subgame-perfect strategies such that the monetary authority validates the private-sector expectations $\hat{\pi}_t=\pi_t$. These strategies must specify a plan of action for every possible monetary authority action, and each of these plans must be sustainable.

To characterize an action as sustainable seems potentially complicated. It is possible, though, to easily characterize sustainable actions in terms of continuation values: for an action π to be sustainable, there should be a schedule of continuation values in $\{V(s)\}_{s\in S}$ such that the monetary authority validates the private-sector expectations $\hat{\pi}=\pi$.

Definition 6 *An action $\pi\in\Pi$ is a sustainable action at $s\in S$ if there exists vectors $w_1,w_2\in\{V(s')\}_{s'\in S}$ such that*

$$\begin{aligned}&u\left(y^*(s)\right)-g(\pi)+\delta\sum_{s'}\mu\left(s'|s\right)w_1\left(s'\right)\\ \geq\;&u\left(y\left(\pi'-\pi;s\right)\right)-g\left(\pi'\right)+\delta\sum_{s'}\mu\left(s'|s\right)w_2\left(s'\right)\end{aligned}$$

for all $\pi' \geq \delta$.

Vector w_1 details the continuation value for each state s' if the monetary authority validates the private-sector expectations; loosely speaking, it is the "reward." The "punishment" vector w_2 states the continuation value if the monetary authority does not validate the private-sector expectations. Because continuation values belong to $\{V(s)\}_{s \in S}$, they correspond to some SE so there exist sub-game perfect strategies sustaining the action π. Note also the sustainable actions are defined with a simple punishment rule: any deviation π' has the same punishment, i.e., the same continuation value vector w_2. These simple punishment rules are without loss of generality, as first pointed out in Abreu (1988).

The next theorem states the key results from Abreu et al. (1990) used in this paper.

Theorem 1 *For all $s \in S$, $V(s)$ is a non-empty and compact set in $\Re$. Moreover, for any sequential equilibrium $\{\sigma, \hat{\sigma}, F, c\}$ and for all nodes $h^{t-1} \in H^{t-1}$, $t \geq 1$,*

1. *The private-sector welfare satisfies*

$$v\left(c; h^j, F\right) \in V(s)$$

   almost everywhere in $H^j | h^{t-1}$, $j \geq t-1$;

2. *Strategy σ specifies sustainable actions almost everywhere $X^j | h^{t-1}$, $j \geq t-1$.*

Finally, for any profile of sustainable actions $\{\pi(s)\}_{s \in S}$ and a date $t \geq 1$, there exists a sequential equilibrium such that F assigns probability one to $\{\pi(s)\}_{s \in S}$ at date t.

Proof. See Abreu et al. (1990). For a simpler proof restricted to perfect monitoring games such as the present one, see Cronshaw and Luenberger (1994) or Ljungqvist and Sargent (2000) ■

It is useful to rephrase Theorem 1 above in terms of sustainable plans. First, it implies that a sustainable plan M assigns positive probability only to sustainable actions. Moreover, for a given profile of sustainable actions and a given date, there exists at least one sustainable plan implementing these actions at the given date with probability 1. However, Theorem 1 does not imply that any probability measure M over sustainable actions is sustainable.

Everything is in place to characterize the BSE and the most important result of this section. Using the results in Theorem 1, I show that the BSE features the lowest sustainable inflation rate along the whole equilibrium path. Hence, while strategies are history dependent in a potentially complicated way, the best sustainable plan M^* is unique and strikingly simple: along the equilibrium path inflation depends only on the current state s.

Proposition 1 *A sequential equilibrium is a best sequential equilibrium if and only if* $\sigma(x^t) = \pi^*(s_t)$ *almost everywhere in* X^t *for all* $t \geq 1$ *where*

$$\pi^*(s) = \inf\{\pi | \pi \textit{ is sustainable at } s\}.$$

Proof. Using the fact that $V(s)$ is compact, π is sustainable if and only if

$$\delta \sum_{s'} \mu(s'|s)(\sup\{V(s')\} - \inf\{V(s')\}) \geq u(y(\pi' - \pi; s)) - u(y^*(s)) - (g(\pi') - g(\pi)) \quad (8)$$

for all $\pi' \geq \delta$.

I will use this to prove that the set of sustainable actions is non-empty and compact for all $s \in S$. First, the set always includes any one-period Nash equilibrium and (8) follows trivially. Consider a convergent sequence $\{\pi_j\}_{j=0}^{\infty} \to \tilde{\pi}$ of sustainable actions at s. Since the right-hand side of 8 is a measurable function for all $\pi' \geq \delta$, it follows that $\tilde{\pi}$ satisfies (8). Hence $\pi^*(s)$ exists and it is a sustainable action at s.

The second part of the proof shows that $\pi^*(s)$ effectively characterizes the BSE. First, I prove that $V(s)$ is bounded above by

$$\bar{v}(s) = u(y(\pi^*(s))) - g(\pi^*(s)) + \delta \sum_{s'} \mu(s'|s_1) \sup\{V(s')\}.$$

To prove this, assume there exists a SE with strictly larger value $v\left(\tilde{c}; s_1, \tilde{F}\right) > \bar{v}(s_1)$ for $s_1 \in S$. Because all actions with positive probability in a SE are sustainable actions, $v\left(\tilde{c}; h^2, \tilde{F}\right) \leq \sup\{V(s_2)\}$ almost everywhere in H^2. Hence $v\left(\tilde{c}; s_1, \tilde{F}\right) > \bar{v}(s_1)$ implies that with positive probability

$$u(y(\pi^*(s_1))) - g(\pi^*(s_1)) < u(y(\pi^*(s_1))) - g\left(\tilde{\pi}\left(h^1\right)\right)$$

and thus $g(\tilde{\pi}(h^1)) < g(\pi^*(s_1))$. But $g(\pi)$ is strictly increasing in π. Therefore it would imply that there exists a sustainable inflation rate $\tilde{\pi}(h_1)$ strictly lower than $\pi^*(s_1)$, which violates the definition of $\pi^*(s_1)$.

It follows that $\bar{v}(s_1) = \sup\{V(s_1)\}$—from Theorem 1 $\bar{v}(s_1) \in V(s_1)$ as there exists a SE featuring $\pi^*(s_1)$ at date $t = 1$.

The same construct for any $s \in S$ implies

$$\sup\{V(s)\} = u(y^*(s)) - g(\pi^*(s)) + \delta \sum_{s'} \mu(s'|s) \sup\{V(s')\} \tag{9}$$

for all $s \in S$. The system of equations described by (9) proves the if and only if part of the proposition. ■

Next I present a simple corollary of Proposition 1 that plays an important part in the result. It implies that the BSE must be sustained by promising the highest reward and threatening with the utmost punishment. Hence, if at date $t = 1$ the best sustainable inflation $\pi^*(s_1)$ is achieved for all realizations of s_1, it must be that the BSE follows. The corollary highlights the importance of the recursive structure of the SE.[29]

[29] To be precise, it is an application of the bang-bang result discussed in Abreu et al. (1990).

Corollary 1 *If $\pi^*(s) > \delta$, a sequential equilibrium with $\sigma(x^1) = \pi^*(s_1)$ almost everywhere in X^1 is a best sequential equilibrium.*

Proof. Assume there exists a SE $\left\{\tilde{\sigma}, \tilde{\sigma}', \tilde{c}, \tilde{F}\right\}$ with $\tilde{\sigma}(x^1) = \pi^*(s_1)$ a.e. in X^1 but $v\left(\tilde{c}; s_1, \tilde{F}\right) < \sup\{V(s_1)\}$. It then implies that there exists a reward continuation value $w_1$which renders $\pi^*(s_1)$ sustainable with $\delta \sum_{s'} \mu(s'|s_1)(\sup\{V(s')\} - w_1(s')) > 0$. Since $\pi^*(s_1) > \delta$, then there exists $\pi^{**} < \pi^*(s_1)$ satisfying (8) and contradicting the definition of $\pi^*(s_1)$ ■

A.3 Date $t = 0$ Asset Markets

In this section, I introduce trade in nominal and real assets. I lay out a trade structure that provides asset prices but preserves the monetary policy game exactly as described above. Neither payoffs nor action sets are modified; in a precise sense to be made clear later, the equilibrium value set is left intact by asset trade.

A.3.1 Asset Market Environment and Equilibrium Definition

In addition to the economy described above, I introduce a $t = 0$ period when all asset trading takes place. At date $t = 0$, the private sector has an endowment $y_0 > 0$ and there are no monetary decisions.

There are real and nominal assets that are simultaneously traded at date $t = 0$. Available real assets are a full set of contingent claims $b(s^t)$, sold at discount price $q(s^t)$, which deliver one unit of output at node $s^t \in S^t$, $t \geq 1$. There is also a set of nominal claims $B(s^t)$, sold at discount price $Q(s^t)$, which deliver one nominal unit in the event $s^t \in S^t, t \geq 1$. All asset prices are in units of date $t = 0$ consumption. Repayment of all claims is perfectly enforceable. Let $q = \{q(s^t) : s^t \in S^t, t \geq 1\}$ and similarly for Q, b and B.

It is important to note that asset markets are not complete. The history of actions $h^t \in H^t$ introduces income volatility. Yet assets only span exogenous histories S^t. When I discuss the results, it will be clear that the results go through with a more limited asset structure or sequential trading.

The price level $P(h^t)$ at node h^t is given by

$$P\left(h^t\right) = \pi_t P\left(h^{t-1}\right) \tag{10}$$

for all $h^t \in H^t$, $t \geq 1$. As the price level is irrelevant at $t = 0$, $P(h^0) = 1$ solves the nominal indeterminacy. Let $P = \{P(h^t) : h^t \in H^t, t \geq 0\}$.

The monetary authority's net asset holdings, denoted $\{b, B\}$, are exogenously dictated by the government. To avoid any interaction between fiscal and monetary policy, I assume that the government budget is cleared via lump-sum taxes $\tau = \{\tau(h^t) : h^t \in H^t, t \geq 0\}$. Therefore, the transfers satisfy

$$\sum_{t=1}^{\infty} \sum_{s^t \in S^t} \left(q\left(s^t\right) b\left(s^t\right) + Q\left(s^t\right) B\left(s^t\right)\right) \leq \tau_0 \tag{11}$$

and

$$0 \leq b\left(s^t\right) + \frac{B(s^t)}{P(h^t)} + \tau\left(h^t\right) \tag{12}$$

for all $h^t \in H^t$, $t \geq 1$.

The private-sector date $t = 0$ asset market problem is to choose the date $t = 0$ consumption c_0, a consumption plan $c = \{c(h^t) : h^t \in H^t, t \geq 1\}$, and real and nominal assets $\left\{\hat{b}, \hat{B}\right\}$ to solve

$$\max u\left(c_0\right) + \delta \sum_{s_1 \in S_1} \mu\left(s_1 | s_0\right) v\left(c; s_1, F\right) \tag{13}$$

subject to

$$c_0 + \sum_{t=1}^{\infty} \sum_{s^t \in S^t} \left(q\left(s^t\right) \hat{b}\left(s^t\right) + Q\left(s^t\right) \hat{B}\left(s^t\right)\right) \leq y_0 - \tau_0, \tag{14}$$

and for all $h^t \in H^t$, $t \geq 1$,

$$c\left(h^t\right) \leq \hat{b}\left(s^t\right) + \frac{\hat{B}\left(s^t\right)}{P\left(h^t\right)} + y\left(\pi_t, \hat{\pi}_t; s_t\right) - \tau\left(h^t\right) \tag{15}$$

as well as non-negative consumption $c\left(h^t\right) \geq 0$ and borrowing constraints

$$\begin{aligned} \hat{b}\left(s^t\right) &\geq -\bar{b}, \\ \hat{B}\left(s^t\right) &\geq -\bar{B}, \end{aligned}$$

for all $s^t \in S^t$, $t \geq 1$ with $\bar{b} > 0$ and $\bar{B} > 0$. Importantly, the private sector takes as given all asset prices, the monetary authority's actions as well as the lump sum taxes τ.

I will assume that all bonds are in zero aggregate supply. Asset market clearing then implies that for all $s^t \in S^t$, $t \geq 1$,

$$\begin{aligned} B\left(s^t\right) + \hat{B}\left(s^t\right) &= 0, \\ b\left(s^t\right) + \hat{b}\left(s^t\right) &= 0. \end{aligned}$$

I also assume that the monetary net asset holdings $\{b, B\}$ never bind the private-sector borrowing constraints.

A date $t = 0$ asset market equilibrium combines a SE, exactly as defined above, with optimal portfolio and market clearing conditions. The key output is the nominal asset prices.

Definition 7 *A date $t = 0$ asset market equilibrium (AME) consists of asset prices $\{q, Q\}$, a price level path P, lump-sum taxes τ, asset holdings $\left\{b, \hat{b}, B, \hat{B}\right\}$, date $t = 0$ consumption c_0, and a sequential equilibrium $\{\sigma, \hat{\sigma}, F, c\}$ such that*

1. *assets $\left\{\hat{b}, \hat{B}\right\}$ and consumption decisions $\{c_0, c\}$ solve the private-sector date $t = 0$ asset market problem, (13);*

2. *date $t = 0$ consumption is feasible $c_0 \leq y_0$,*

3. prices $\{q, Q\}$ clear all asset markets,

4. lump-sum taxes satisfy (11) and (12),

5. the price level is given by (10) for all $h^t \in H^t$, $t \geq 0$.

I have not considered the possibility that the private sector holds monetary balances, so the nominal interest rate could be negative. To avoid this, I assume that the lowest sustainable inflation $\pi^*(s)$ and the real interest rate are non-negative.

Condition 1 *For all $s \in S$,*

$$\pi^*(s)\left(\delta \frac{u^c(y^*(s))}{u^c(y_0)}\right)^{-1} \geq 1.$$

A.4 Asset Prices and the Best Sustainable Equilibrium

In this section I analyze the mapping between two components of a date $t = 0$ asset market equilibrium: the asset prices and the sustainable equilibrium. This mapping is found to be a correspondence, but the key result is actually characterized by an exception: the relationship between the short-term nominal interest rate and the best sustainable equilibrium is one-to-one.

I start by proving the claim that asset trading has not changed the equilibrium value set. Of course, I disregard the trivial difference induced by the inclusion of period $t = 0$. I define the equilibrium value set for date $t = 0$ asset market equilibrium (AME) as

$$\tilde{V} = \{v(c; F) \mid \{F, c\} \text{ are part of a AME}\}.$$

By definition, an AME contains a SE; thus $\tilde{V} \subset V$. Proposition 2 proves that $V \subset \tilde{V}$ as well.

Proposition 2 *Let $\{\sigma, \hat{\sigma}, F, c\}$ be a sequential equilibrium. Then there exists a unique date $t = 0$ market equilibrium with $\{\sigma, \hat{\sigma}, F, c\}$.*

Proof. Given a SE, the price level is trivially given by (10). Private-sector bond holdings are given from the market clearing conditions, $\hat{b} = -b$ and $\hat{B} = -B$, and the lump-sum taxes are solved from the government budget constraints (11) and (12). The only non-trivial objects in a candidate AME are asset prices. The necessary first-order conditions associated with (13) are

$$\begin{aligned} u^c(c_0)\, q\left(s^t\right) &= \delta^t \mu\left(s^t | s_0\right) \int_{H^t} u^c\left(c\left(h^t\right)\right) F\left(dh^t | s^t\right), \\ u^c(c_0)\, Q\left(s^t\right) &= \delta^t \mu\left(s^t | s_0\right) \int_{H^t} \frac{u^c\left(c\left(h^t\right)\right)}{P\left(h^t\right)} F\left(dh^t | s^t\right) \end{aligned}$$

for all $s^t \in S^t$, $t \geq 1$. The only candidate for c_0 is y_0, as imposed by feasibility. Hence, for a given SE, asset prices are uniquely pinned down by the above necessary first-order conditions. Since $P\left(h^t\right) > 0$ for all $h^t \in H^t$, and $u^c(c) > 0$, asset prices are positive. Finally, consumption decisions satisfy the household budget constraints by Walras' Law ■

Asset prices in an AME are characterized by the necessary first-order conditions associated with (13). Since a SE requires $\pi_t = \hat{\pi}_t$ almost everywhere, $c\left(h^t\right) = y^*\left(s_t\right)$ with probability one along the equilibrium path. Therefore real and nominal asset prices satisfy

$$q\left(s^t\right) = \delta^t \mu\left(s^t | s_0\right) \frac{u^c\left(y^*\left(s_t\right)\right)}{u^c\left(y_0\right)}, \tag{16}$$

$$Q\left(s^t\right) = \delta^t \mu\left(s^t | s_0\right) \frac{u^c\left(y^*\left(s_t\right)\right)}{u^c\left(y_0\right)} \int_{H^t} \frac{1}{P\left(h^t\right)} F\left(dh^t | s^t\right) \tag{17}$$

for all $s^t \in S^t$, $t \geq 1$. Thus the real interest rate is exogenous. The price level $P\left(h^t\right)$ is a function only of π^t. This allows writing asset prices in terms of the associated sustainable plan M. It is useful to combine the asset price conditions:

$$Q\left(s^t\right) = q\left(s^t\right) \int_{\Pi^t} \frac{1}{P\left(h^t\right)} M\left(d\pi^t | s^t\right). \tag{18}$$

Finally I price a short-term nominal bond. It pays one nominal unit in all states of the world at date $t = 1$, and thus its discount price is

$$R_0^{-1} = \sum_{s_1 \in S} Q(s_1).$$

The two key outputs from an AME are the sustainable plan and the asset prices. I want to analyze the mapping between these two equilibrium objects. It turns out to be convenient to think of the mapping in terms of value sets. Let

$$\Gamma\left(\{q, Q\}\right) = \{v\left(c; F\right) \mid \{F, c, q, Q\} \text{ are a part of an AME}\}$$

defined over any positive asset price schedule $\{q, Q\}$. Proposition 2 has established that, given a sustainable plan M, there exists a unique set of asset prices that constitute an AME generating M. However, the converse is not true. Given asset prices $\{q, Q, \}$ there may be no SE conforming to an AME. Or there may be multiple SE. This is important in order to understand the results on coordination.

First I show that there may be no AME for given asset prices $\{q, Q\}$. An extreme example is convenient. Assume there is a unique one-period Nash equilibrium, $\tilde{\pi}$, and, for simplicity, assume it is state invariant. It can be shown that, for low enough $\delta > 0$, only $\tilde{\pi}$ is a sustainable action and hence all SEs must feature $\tilde{\pi}$ with probability one. Therefore, for any asset prices $\{q, Q\}$ such that

$$Q\left(s^t\right) \neq q\left(s^t\right) \frac{1}{\tilde{\pi}^t}$$

there would be no AME with such prices. In this example, $\Gamma\left(\{q, Q\}\right) = \varnothing$ for all but one price schedule.

Next I consider the possibility of multiple SEs associated with given asset prices $\{q, Q\}$. For simplicity there is no exogenous uncertainty $s_t = \bar{s}$ for all $t \geq 1$, with $y^*\left(\bar{s}\right) = y_0$.

Assume that there exists a SE $\{\sigma, \hat{\sigma}, F, c\}$ that places probability one in history $\tilde{h}^t = \{\tilde{\pi}\}^t$ for all $t \geq 0$ and its value lies in the interior of $V(\bar{s})$. Then there exists a SE $\{\sigma', \hat{\sigma}', F', c'\}$ that places equal probability to histories $\{\tilde{\pi}+\varepsilon_1, \tilde{\pi}, \tilde{\pi}, ..\}$ and $\{\tilde{\pi}-\varepsilon_2, \tilde{\pi}, \tilde{\pi}, ..\}$ with

$$\frac{1/2}{\tilde{\pi}+\varepsilon_1} + \frac{1/2}{\tilde{\pi}-\varepsilon_2} = \frac{1}{\tilde{\pi}}$$

and ε_1 small enough. That $\{\sigma', \hat{\sigma}', F', c'\}$ is indeed a SE follows from the fact that, for an arbitrarily small ε_1, both $\tilde{\pi}+\varepsilon_1$ and $\tilde{\pi}-\varepsilon_2$ are sustainable actions. As long as the costs of inflation $g(\pi)$ have some curvature, the two SEs have different ex-ante welfare.

By construction, both SEs have the same asset prices,

$$\begin{aligned} q(s) &= \delta, \\ Q(s) &= \delta \frac{1}{\tilde{\pi}} = \delta \left(\frac{1/2}{\tilde{\pi}+\varepsilon_1} + \frac{1/2}{\tilde{\pi}-\varepsilon_2} \right). \end{aligned}$$

Hence these asset prices $\{q, Q\}$ do not determine the ex-ante welfare level of an AME. Hence, $\Gamma(\{q, Q\})$ is not a singleton.

Hence the mapping from date $t = 0$ market equilibrium asset prices to sustainable plans is a correspondence, $\Gamma : \{q, Q\} \rightrightarrows \{V, \emptyset\}$. However, asset prices are informative to some extent. A key observation is that the arbitrage condition (18) implies a one-to-one mapping between asset prices and the following moment of the distribution of the price level,

$$q(s^t)/Q(s^t) = \left[\int_{\Pi^t} \frac{1}{P(h^t)} M\left(d\pi^t | s^t\right) \right]^{-1}.$$

Many SEs or no SE may share this moment, yet it is unlikely that all SEs do. In particular, the moment implies that asset prices constrain the support of the compatible sustainable price levels. For example, a sustainable plan assigning probability one to prices strictly above or strictly below $q(s^t)/Q(s^t)$ will not satisfy the moment.

The next is the key result of the paper. As I just discussed, the mapping between asset prices and sustainable policy plans is generally a correspondence. However, the exception is of utmost interest: a date $t = 0$ market equilibrium features the best sustainable equilibrium if and only if the one-period nominal bond is priced at a certain discount rate.

Theorem 2 *A date $t = 0$ asset market equilibrium features a best sequential equilibrium if and only if*

$$R_0^{-1} = \delta \sum_{s_1 \in S} \mu(s_1|s_0) \frac{u^c(y^*(s_1))}{u^c(y_0)} \frac{1}{\pi^*(s_1)}. \tag{19}$$

Proof. If an AME is a BSE, then (17) implies $Q(s_1) = \delta\mu(s_1|s_0) \frac{u^c(y^*(s_1))}{u^c(y_0)} \frac{1}{\pi^*(s_1)}$ for each $s_1 \in S$ and (19) follows trivially.

Consider now a AME satisfying (19). The arbitrage condition (18) implies that

$$\sum_{s_1 \in S} Q(s_1) = \sum_{s_1 \in S} q(s_1) \left(\int_\Pi \frac{1}{P(h^1)} F(d\pi_1|s_1) \right)$$

and combined with (19)

$$\sum_{s_1 \in S} q(s_1) \left(\frac{1}{\pi^*(s_1)} - \int_\Pi \frac{1}{P(h^1)} F(d\pi_1|s_1) \right) = 0.$$

By the definition of π^*, $P(h^1) \geq \pi^*(s_1)$ for any SE. Therefore

$$\frac{1}{\pi^*(s_1)} - \int_\Pi \frac{1}{P(h^1)} F(d\pi_1|s_1) \geq 0$$

for each $s_1 \in S$. It follows that $P(h^1) = \pi^*(s_1)$ for all $s_1 \in S$. Finally, Condition 1 implies that Corollary 1 applies and the only SE compatible with (19) is the BSE ■

The first thing to note about Theorem 2 is that the price of a single asset for a single date is sufficient to pin down the BSE for all periods. This stands in contrast with practically any other *complete* price schedule, i.e., it is not possible to pin down uniquely other SEs even if all asset prices can be determined. This strong result is built on the properties of the BSE summarized in Proposition 1. In particular, the BSE belongs to the boundary of both the value set V and the set of sustainable actions. It is straightforward to show that the best sequential equilibrium requires the highest discount rate for the short-term nominal interest rate.

Corollary 2 *For any date $t = 0$ asset market equilibrium,*

$$R_0^{-1} \leq \delta \sum_{s_1 \in S} \mu\left(s_1 | s_0\right) \frac{u^c\left(y^*\left(s_1\right)\right)}{u^c\left(y_0\right)} \frac{1}{\pi^*\left(s_1\right)}$$

The proof of Theorem 2 applies almost step-by-step to other asset trade arrangements. If only the short-term nominal bond were available, then (19) is exactly the associated necessary first-order condition. Asset trade could also be sequential with no change in the logic of the result. Of course, if asset holdings do change the monetary authority's incentives, then asset prices would also bring cooperative considerations.

It is important to note that Condition 1 is in effect: Theorem 2 does not apply if the zero nominal interest rate bound is binding for the BSE at date $t = 1$. It is easy to find parameters such that (19) would imply a negative interest rate. Interestingly, setting $\sum_{s_1 \in S} Q\left(s_1\right) = 1$ does not solve the problem. Because I use Corollary 1 to prove the result, if the BSE is the optimal monetary policy, there could be other sustainable plans that start at the zero nominal interest rate level but inflation eventually takes off.